Angular Kinematics

Average Angular Velocity

$$\bar{\omega} = \frac{\theta_{final} - \theta_{initial}}{time}$$

Instantaneous Angular Velocity

$$\omega = \frac{d\theta}{dt}$$

Average Angular Acceleration

$$\bar{\alpha} = \frac{\omega_{final} - \omega_{initial}}{time}$$

Instantaneous Angular Acceleration

$$\alpha = \frac{d\omega}{dt}$$

Constant Angular Acceleration Equations

$${}_f = {}_i + \quad t$$

$${}_f^2 = {}_i^2 + \qquad ({}_f - {}_i)$$

$${}_f = {}_i + {}_i t + \tfrac{1}{2} \quad t^2$$

$${}_f = {}_i + \tfrac{1}{2}({}_i + {}_f)$$

Where:

${}_i$ = initial angular position
${}_f$ = final angular position
${}_i$ = initial angular velocity
${}_f$ = final angular velocity
= angular acceleration
t = duration in seconds

Example: How many revolutions will a person complete after initiating a spin at 5 revolutions per second and accelerating at the rate of 2 r/s^2 for 3 seconds?

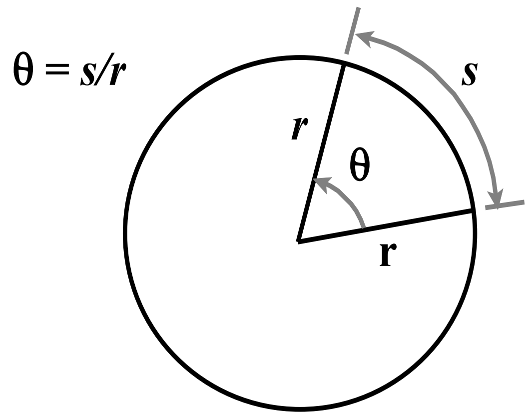
$${}_f = 0 + 5\,(3) + \tfrac{1}{2}\,(2)\,3^2 = 15 + 9 = 24.0 \text{ [revolutions]}$$

Note, keep all angular units within an equation in the same units, i.e., degrees (deg), radians(rad) or revolutions (r).

Angular Kinematics

Radian Measure:

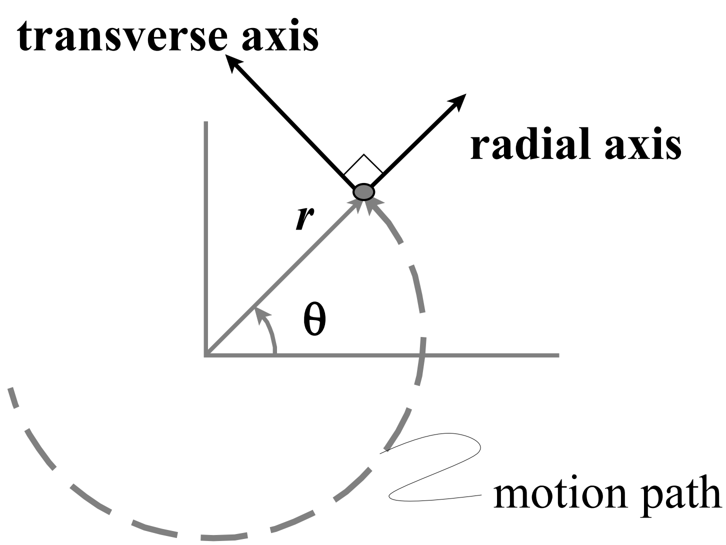
r = **radius**
s = **arc length**
  = **angle**



When $s = r$, $\theta = 1$ radian

Since,   **= *s* / *r***
therefore,   ***s* = *r*** , where   is in radians

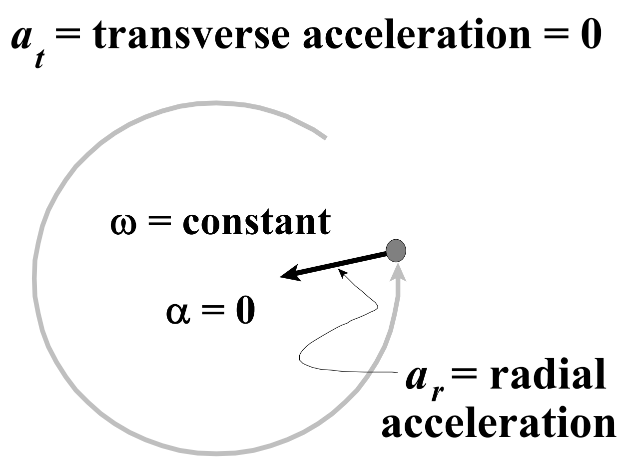
Radial and Transverse Axes:



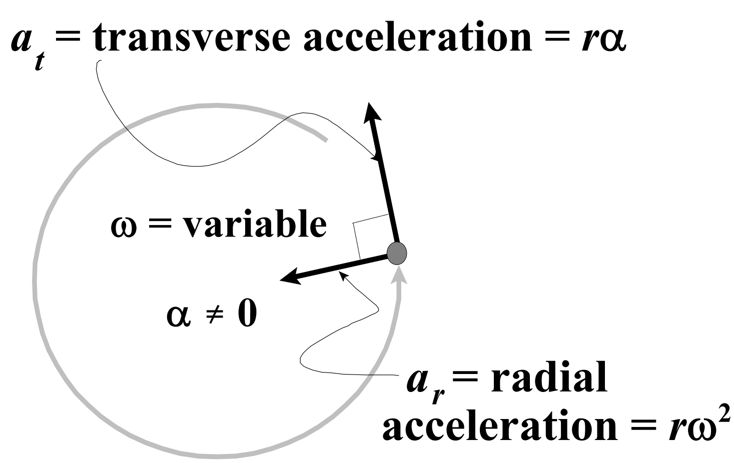
- **radial axis** begins at the point and is directed along the line from the origin to the point
- **transverse axis** is orthogonal to radial (i.e., +90 degrees rotation)

Angular Kinematics

Constant Angular Velocity (= 0):



Variable Angular Velocity (≠ 0):



Relationships between Linear and Angular Kinematics

For Circular Motion:

$= s_{transverse} / r$ or

$s_{transverse} = r$ and

$s_{radial} = r$

Furthermore, $v_{tranverse} = r$

$v_{radial} = 0$ (since r is constant)

and $a_{transverse} = r$

($a_{transverse}$ occurs when angular speed changes)

$a_{radial} = v_{transverse}{}^{2} / r = r \quad {}^{2}$

(a_{radial} is due to directional changes)

$a_{total} = \sqrt{(a_{radial}^2 + a_{transverse}^2}$

(a_{total} is the magnitude of the resultant acceleration)

Example:

A bucket is swung in a circular path with an angular acceleration of 20.0 rad/s^2 at a radius of 1.250 cm. What is the linear velocity and acceleration when the bucket reaches an angular velocity of 25.0 rad/s?

$v_{transverse} = r \quad = 1.250\text{ m} \times 25.0\text{ rad/s} = 31.25\text{ m/s}$
$v_{radial} = 0.0$

Thus, the linear velocity vector is: $\underline{\boldsymbol{v}}$ = (0.00, 31.3) m/s

$a_{transverse} = r \quad = 1.250\text{ m} \times 20.0\text{ rad/s}^2 = 25.0\text{ m/s}^2$
$a_{radial} = r \quad ^2 = 1.250\text{ m} \times (25.0\text{ rad/s})^2 = 781.25\text{ m/s}^2$
or $a_{radial} = v_{transverse}{}^2 / r = 31.25^2 / 1.250 = 781.25\text{ m/s}^2$

Thus, the linear acceleration vector is: $\underline{\boldsymbol{a}}$ = (–781, 25.0) m/s^2