

# Basic Concepts: Space

## Space:

- a boundless three-dimensional volume
- concept that a body occupies a volume
- a particle may be located by describing its position with respect to three orthogonal (right-angled) axes and an origin

Measured by the **metre (m)**, also the kilometre (km), millimetre (mm) and for human dimensions the centimetre (cm).

Instruments for measuring include tape measures, micrometers, anthropometers, GPS and cine, photographic, video, CCD, sonic, magnetic and optoelectronic devices.

**Point:** A location in space which occupies no volume, i.e., no length, depth or width; also called a **particle**.

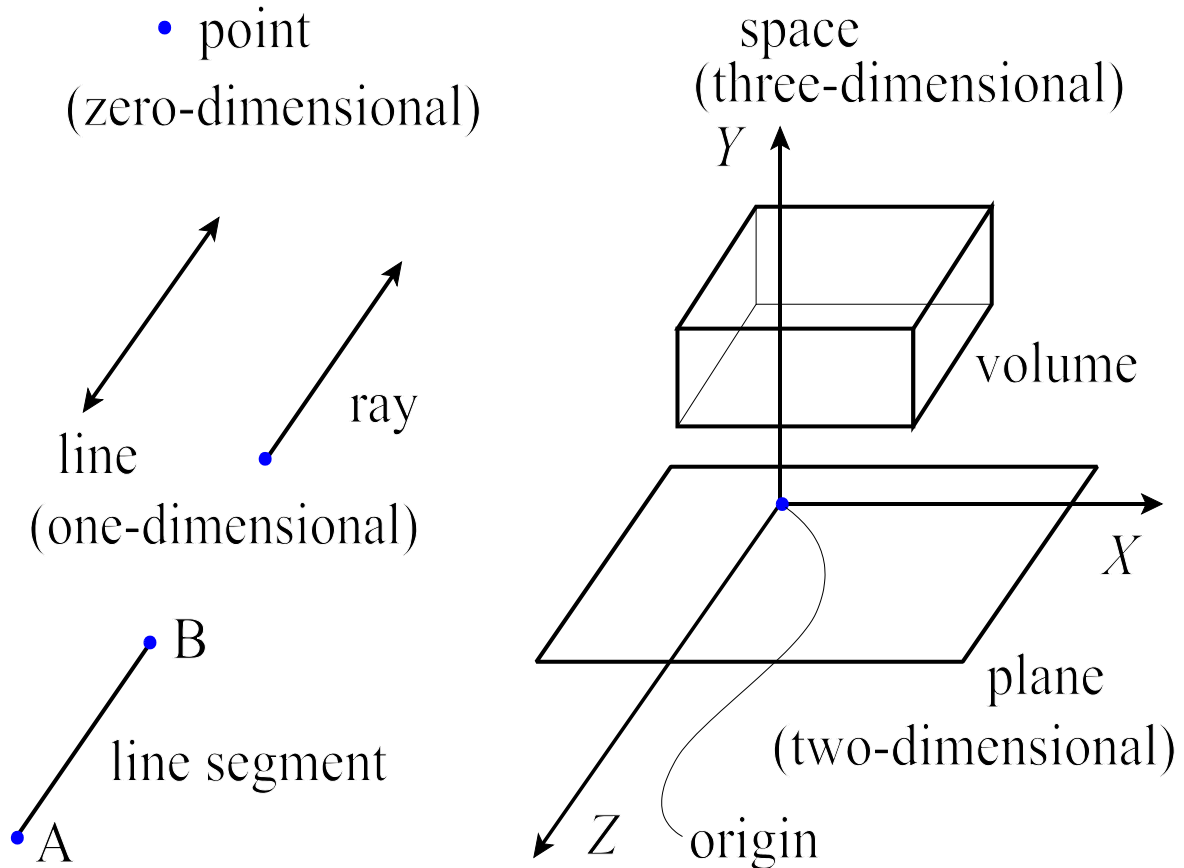
**Line:** An infinite series of points that are “straight” having infinite length and no width or depth. A **ray** starts from a point and extends infinitely in one direction. A **line segment** begins at a point and ends at another point.

**Plane:** A boundless two-dimensional “flat” surface which has no width. E.g., sagittal, mediolateral and anteroposterior planes or horizontal plane.

**Volume:** A bounded three-dimensional space. E.g., cube, sphere, parallelepiped or conic section.

# Spatial Relationships

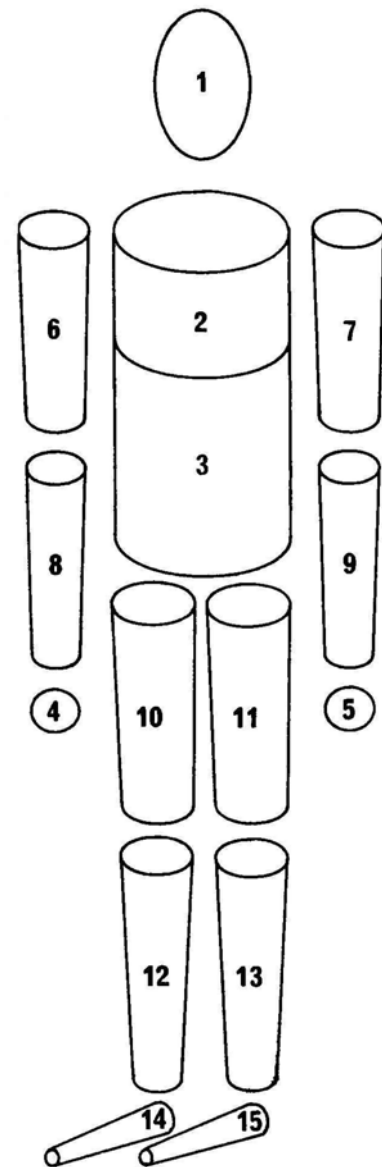
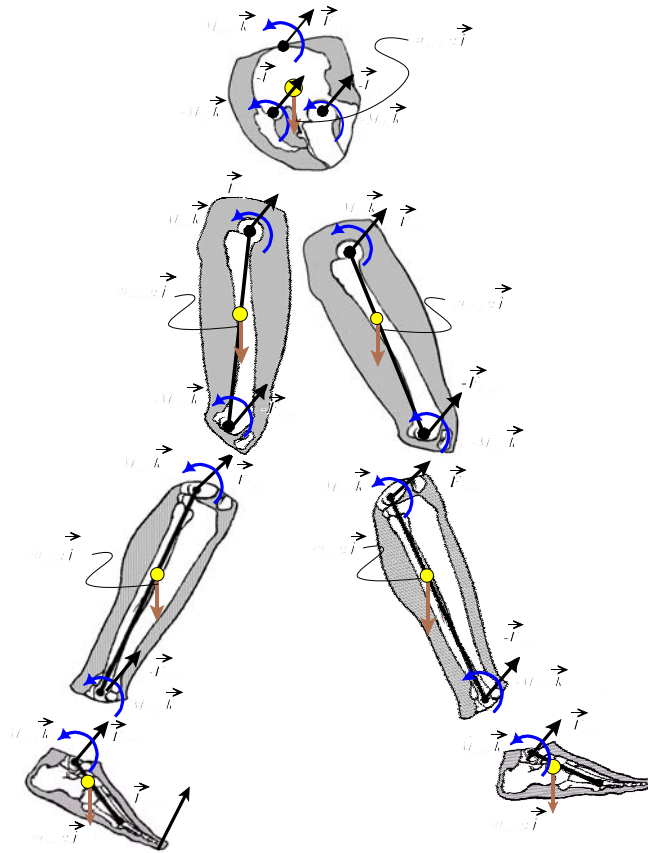
- 0 dimensions: points, particles
- 1 dimension: lines, rays, line segments
- 2 dimensions: planes, segments
- 3 dimensions: volumes, solids



# Two- and Three-dimensional Models of the Human Body

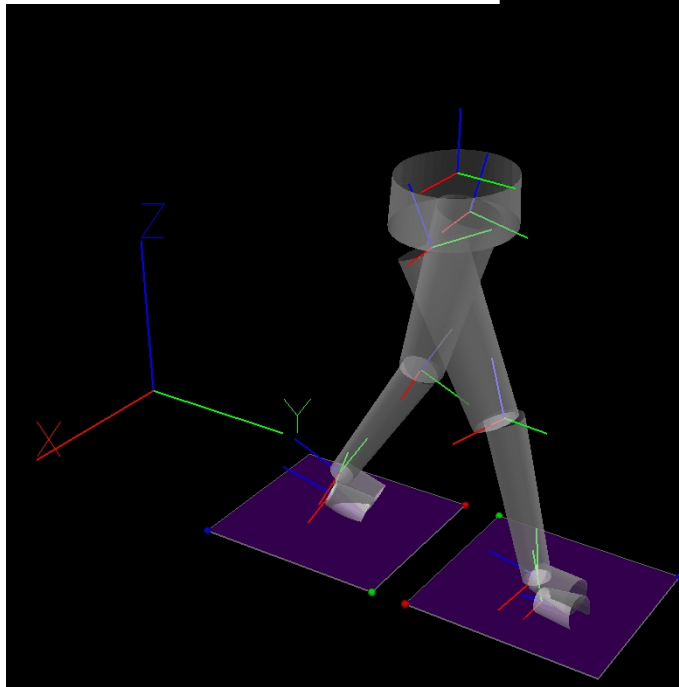
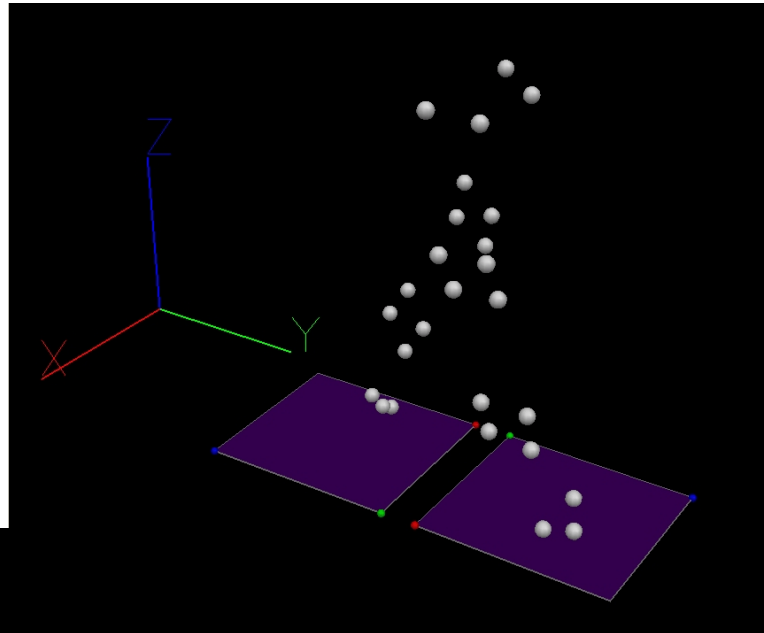
- line segments below represent body segments
- yellow dots represent centres of gravity
- rays represent force vectors

- Hanavan's 15-segment, 3-D model of the human body based on various solids of revolution
- frusta of cones, ellipsoid, elliptical cylinder, spheroids



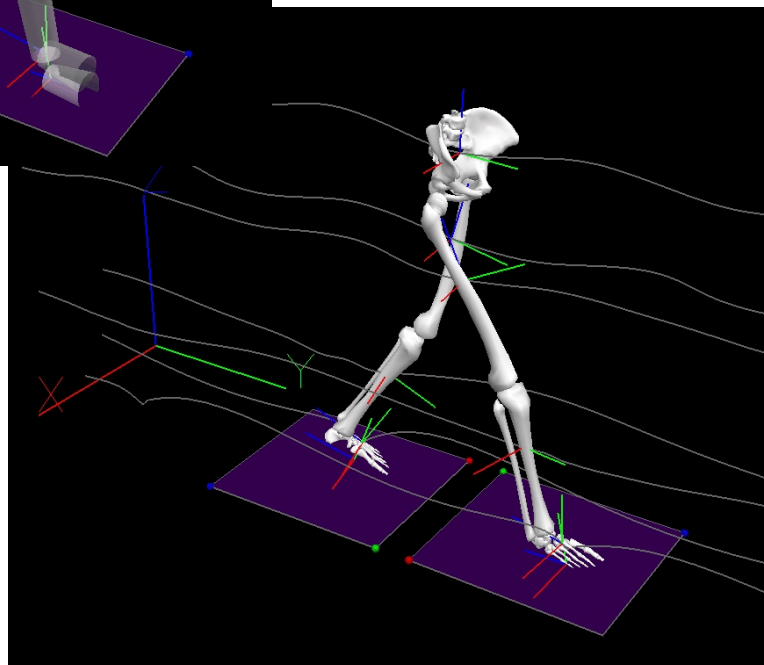
# Three-dimensional Models of the Human Body

- reflective markers attached to skin with at least three per segment



- solids of revolution based of locations of skin markers

- bone models added to reflect human anatomy (not actually used for computations)

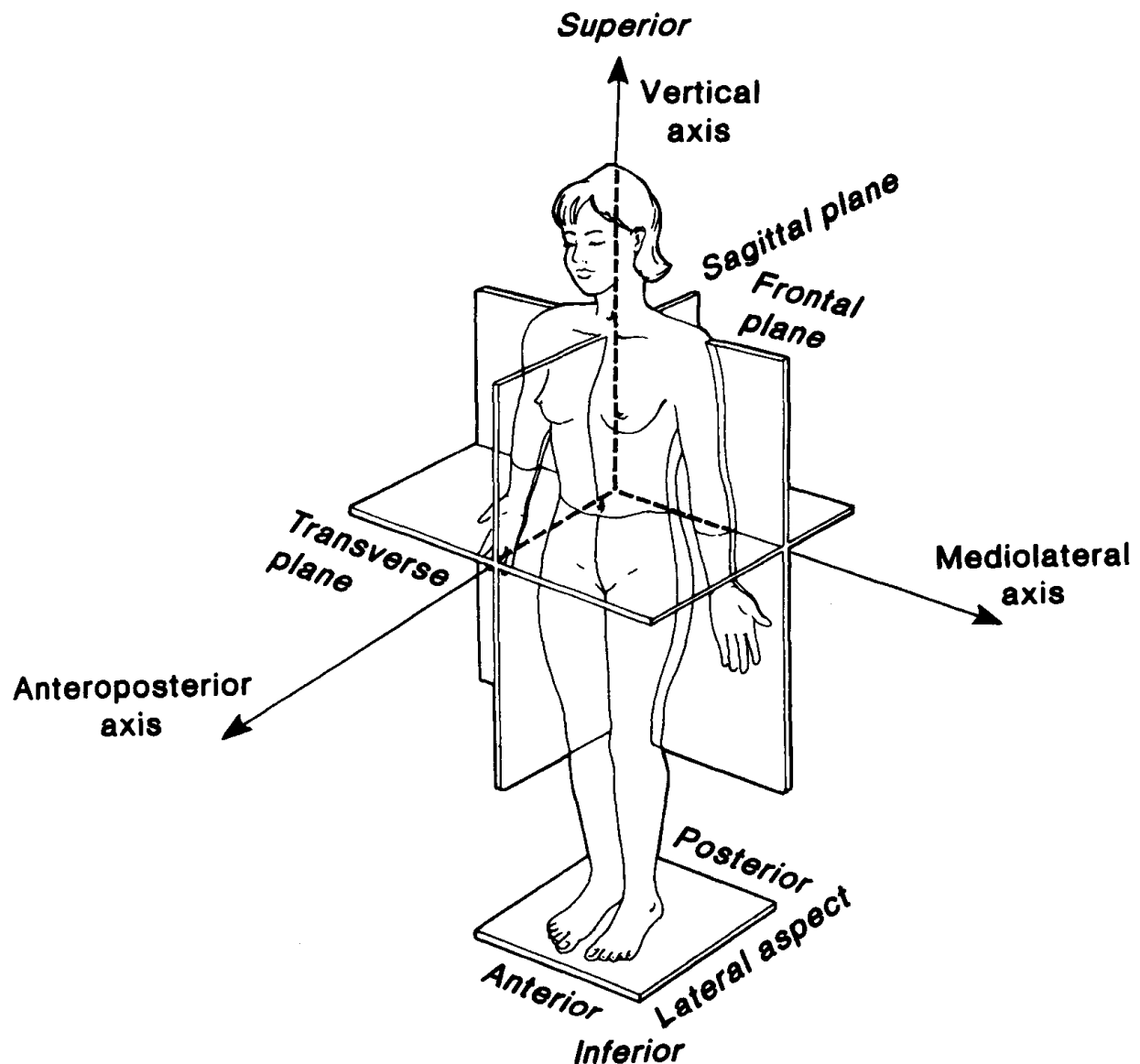


# Frames of Reference

Frames of reference consist of two or three orthogonal (right-angled) axes and a starting point called the **origin**, where the axes intersect.

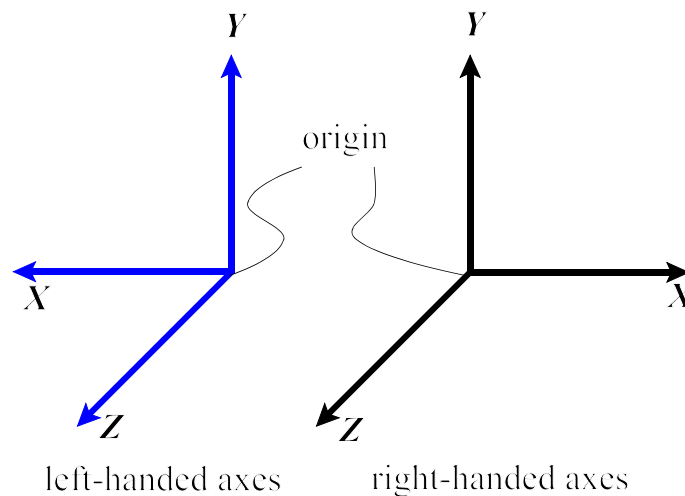
## Cardinal or relative frame of reference

The origin of cardinal axes is usually set at the centre of gravity of the body and moves with the body.



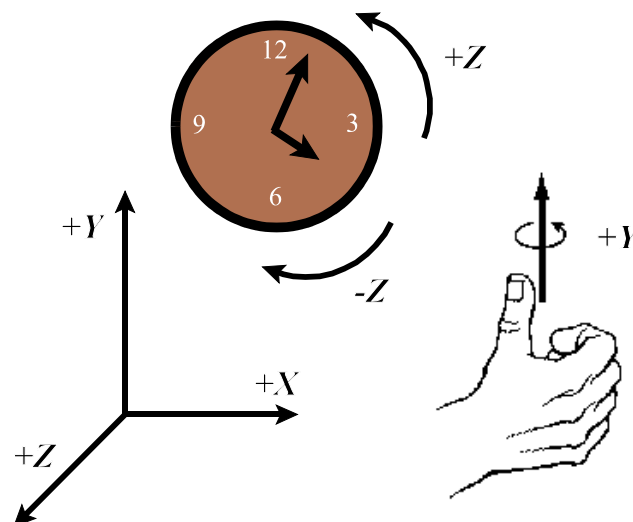
## Newtonian or Absolute Frames of Reference

Origin is usually set at some arbitrary point on the ground and in principle does not move. Axes are also fixed to the ground and do not move. Two **orientations** are possible—right-handed and left-handed. The axes labelling below ( $X$ ,  $Y$ ,  $Z$ ) are the standard order as set by the International Society of Biomechanics (ISB).



### Right-hand rule

The right-handed axis is accepted as the standard orientation. Thus, clockwise rotations are negative.



# Basic Concepts: Mass

## Mass:

1. Fundamental property of a body which characterizes its **resistance to change of motion**, called its **inertia**. A means by which two bodies may be compared based upon their mechanical properties. For example, two bodies in vacuum will accelerate towards the earth in the same manner and offer the same resistance to changes in their translational motion.
2. Mass is also a body's **gravitational potential**. It is the measure of a body's ability to exert gravitational force. In biomechanics, this property is only significant when the body is exceptionally large, such as the earth. Among small bodies, gravitational forces are too small to affect their motions.
3. Mass is also a measure of **energy**, according to Einstein's equation,  $E = m c^2$ . This has little application in biomechanics.

Measured in units of **kilograms (kg)**. Standard kilogram masses are maintained at three sites around the globe. Other standard masses are derived from these three.

Instruments for measuring include balance scales, force transducers and force platforms.

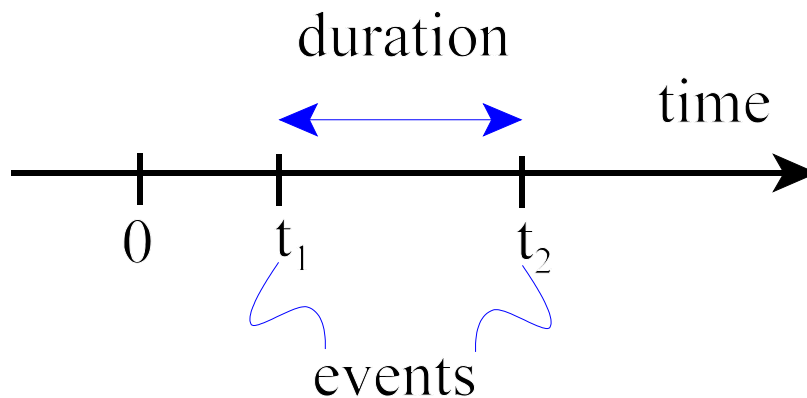
# Basic Concepts: Time

## Time:

The dimension that orders the sequence of events in the universe. A designated instant in time is called an **event**; a period of time is called a **duration** or a **phase**, as in stance phase, airborne phase or push-off phase. The study of the sequence of events or durations between events is called **temporal analysis**.

Measured by the S.I. unit the **second (s)**. Other acceptable units include the millisecond (ms), minute (min), hour (h) and year (a).

Instruments for measuring time include stop watches, stroboscopes, reaction timers, chronographs, phototimers and video tapes (where 1 frame or 2 fields =  $1/30^{\text{th}}$  second).

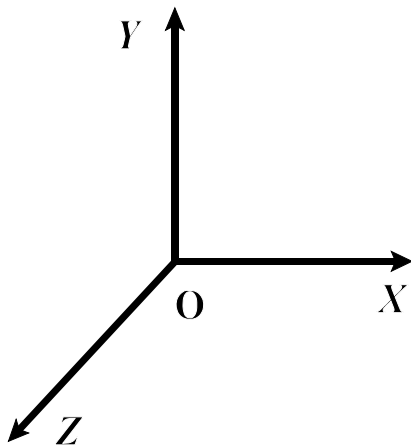


# Newtonian Mechanics

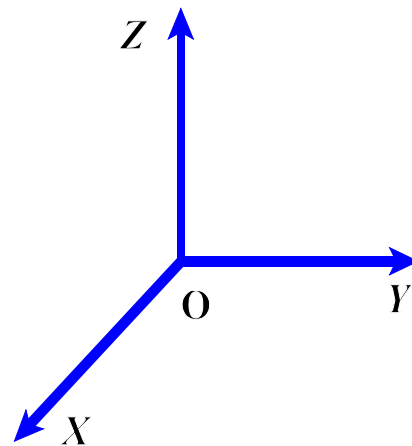
Newtonian mechanics asserts that **space, mass and time are independent** of each other, lines are infinitely straight, mass is unchangeable and time proceeds in uniform steps. Einsteinian or relativistic mechanics has shown these principles to be flawed, however, they may be applied without significant error for most practical purposes (even some impractical ones).

Newtonian mechanics requires an axis whose **origin is fixed in space** and not moving. Such a point does not exist but an origin attached to the ground is sufficiently accurate for most biomechanical purposes.

The following is a typical Newtonian axis system. Positions of bodies are measured with respect to the origin, **O**.



ISB standard labels (used for 2D mechanics)



Alternate axes labels (used in 3D mechanics)

Newton's laws describe the concept of a **force**, which is dependent upon space, mass and time. A force is the action of one body upon another. It is measured in newtons (N) by force platforms or force transducers or indirectly through inverse dynamics. Forces are caused by gravity, electromagnetism or the interaction of particles at the atomic level.

# Scalar and Vectors

**Scalar:** quantity characterized by a magnitude and may have a positive or negative value

Examples: mass, length, time, temperature, work, energy

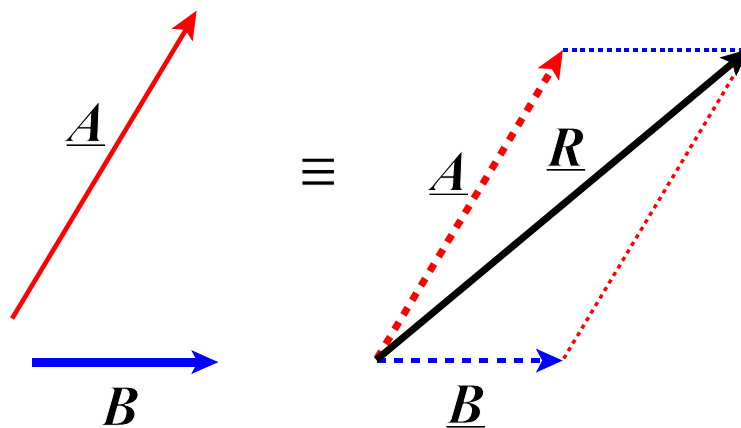
Similar scalar quantities may be added algebraically. I.e.,  $2+3 = 5$ , or the split times of four members of a relay team may be added to obtain their race time or if a 10 kilogram mass is added to a 5 kilogram mass the final mass is 15 kilograms.

**Vector:** quantity that possesses magnitude, direction and adds according to the Parallelogram Law

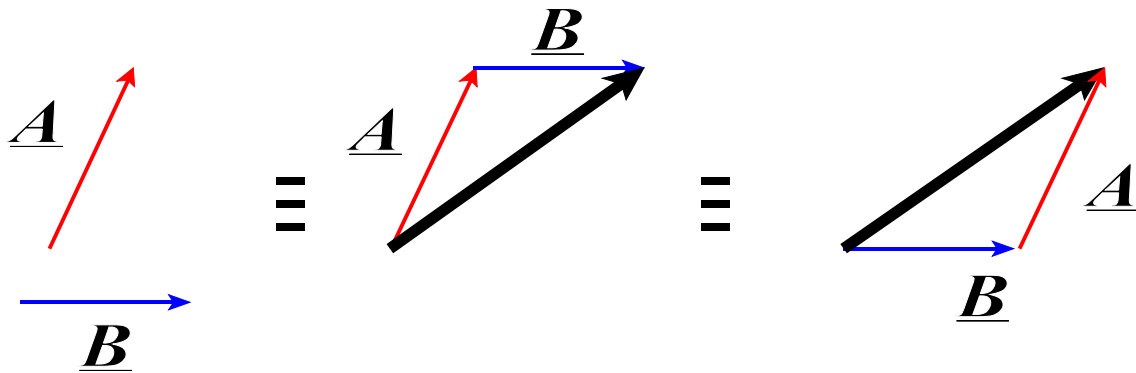
Examples: force, moment of force, impulse and momentum and displacement, velocity and acceleration.

Angular displacements are NOT vectors because they don't follow the Parallelogram Law. Vectors quantities are often indicated by **boldfacing** or by underlining or by vector notation ( $\mathbf{a}_x, \mathbf{a}_y$ ).

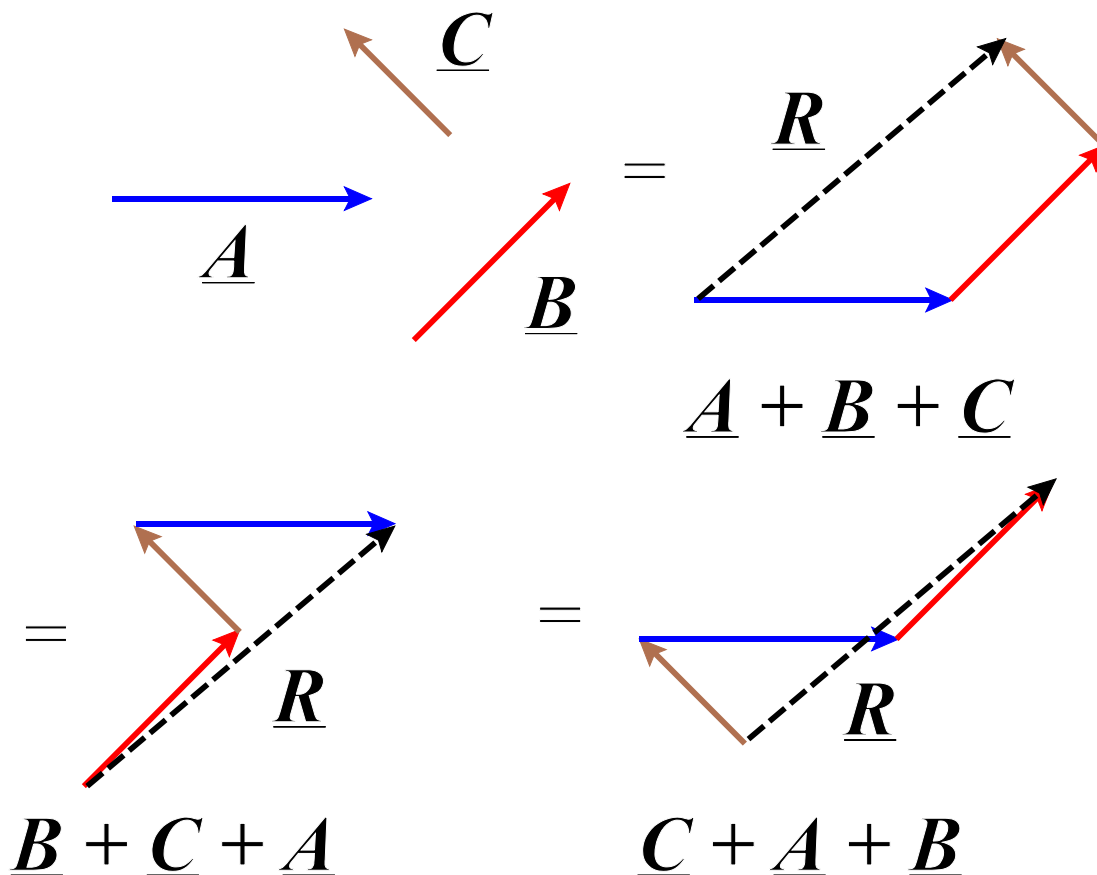
**Parallelogram Law** (also called the commutative law for vector addition) states that the sum of two vectors called the **resultant** is equal to the diagonal formed by the parallelogram that has sides equal to the two vectors.



**Triangle Rule** is a simplified version of parallelogram law

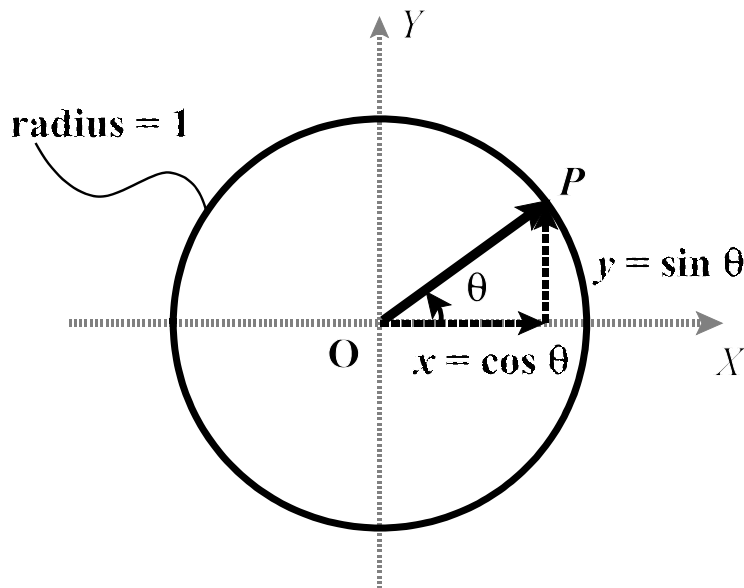


When more than two vectors are added the associative property is applicable. This law states that the vectors may be added together in any order (also called the **Polygon Rule**).



# Trigonometry

Given a circle of radius one unit and point  $P$  on its circumference, the  $x$ -coordinate is defined as the **cosine** of the angle,  $\theta$ , while the  $y$ -coordinate is the **sine** of the angle,  $\theta$ . The slope of the line (i.e.,  $y/x$ ) is called the **tangent** of the angle,  $\theta$ .



As the radius increases the  $x$ - and  $y$ -coordinates of point  $P$  increase proportionally. I.e.,

$$x = \text{radius} \times \text{cosine } \theta$$

and

$$y = \text{radius} \times \text{sine } \theta$$

The ratio of the circumference to the diameter of any circle is equal to a constant, called **pi** ( $\pi$ ). Therefore,

$$\text{circumference} = 2\pi \times \text{radius}$$

# Trigonometry

Given a triangle, the side adjacent to angle  $\theta$  is equal to the length of the hypotenuse times  $\cos \theta$ . The side opposite the angle is equal to the hypotenuse times  $\sin \theta$ .

## Pythagoras' Equation

The hypotenuse of a right-angled triangle is equal to the square root of the sum of squares of the other sides. I.e.,

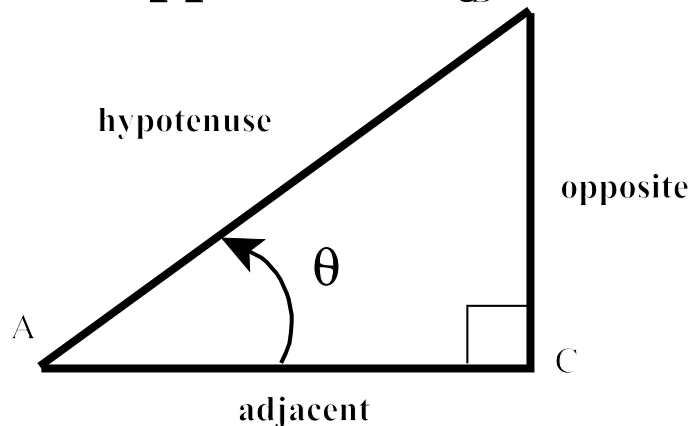
$$\text{hypotenuse} = \sqrt{\text{adjacent}^2 + \text{opposite}^2}$$

## SOH-CAH-TOA (mnemonic)

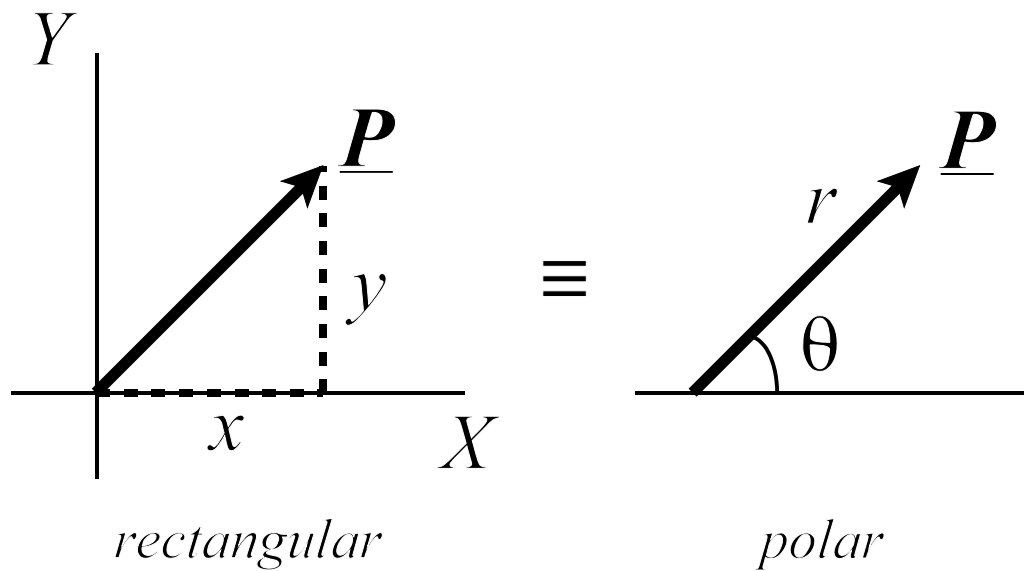
$$\sin \theta = \text{opposite} / \text{hypotenuse}$$

$$\cos \theta = \text{adjacent} / \text{hypotenuse}$$

$$\tan \theta = \text{opposite} / \text{adjacent}$$



# Rectangular and Polar Coordinates



## Rectangular Coordinates:

$$P = (x, y) \quad \text{where} \quad \begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}$$

**Example:** Convert force vector of 25.0 newtons at 35.0 degrees to its rectangular components.

$$\begin{aligned} x &= 25 \cos 35 = 20.5 \text{ N} \\ y &= 25 \sin 35 = 14.34 \text{ N} \end{aligned}$$

Note, compute trig. function first then multiply by radius (25.0).

## Polar Coordinates:

$$P = (r, \theta) \quad \text{where} \quad r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y / x)$$

**Example:** Convert a force vector,  $F = (30.0, 50.0)$  newtons, to its polar coordinates.

$$r = \sqrt{30^2 + 50^2} = 58.3 \text{ N}$$

Note, square first then add the squares. Perform square root operation last.

$$\theta = \tan^{-1}(50 / 30) = 59.0 \text{ deg}$$

Note, on a calculator the **tan**<sup>-1</sup> function is usually obtained by pressing a button labelled **Inv** before pressing the **tan** button.