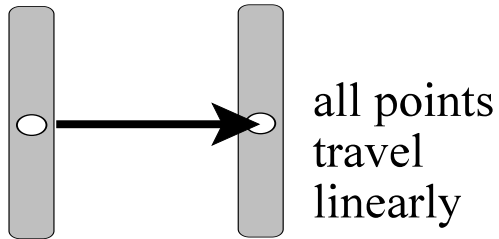


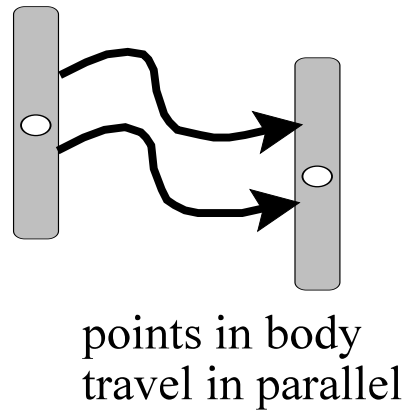
Kinematics

Types of Motion

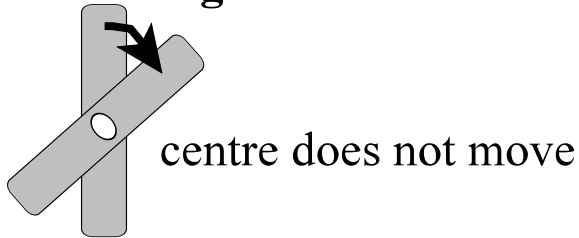
Rectilinear translation or linear motion



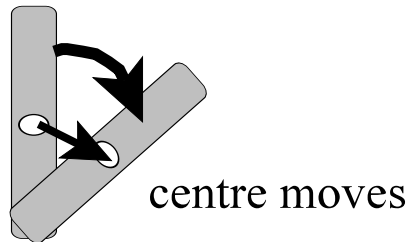
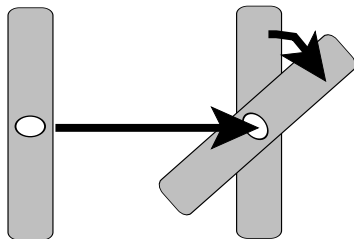
curvilinear translation



Rotation or angular motion

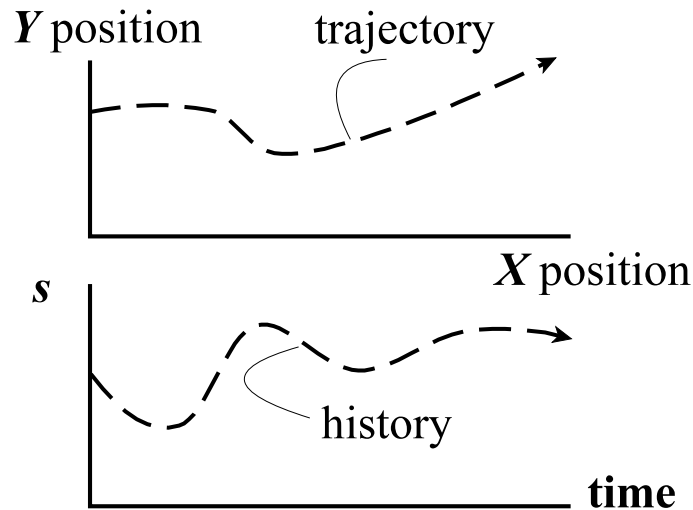


General plane motion = translation and rotation

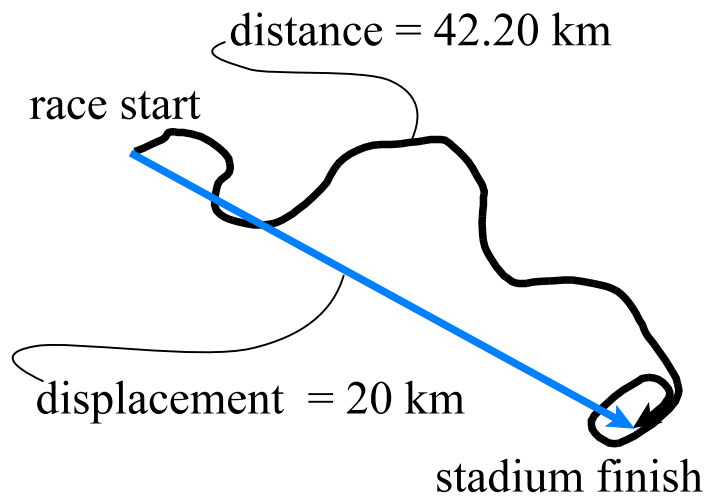


Linear Kinematics

Histories versus Trajectories



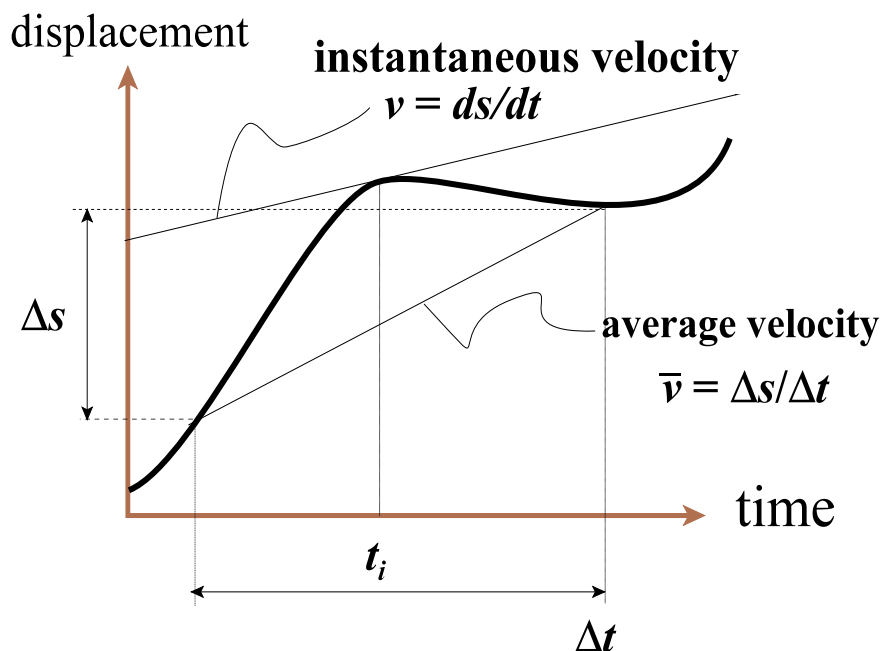
Distance versus Displacement



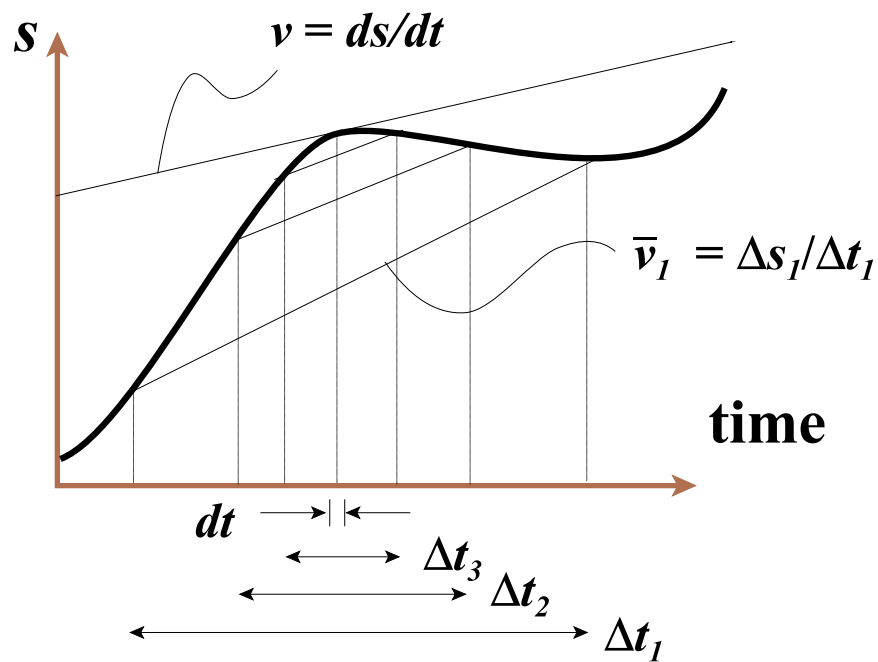
distance = length travelled along a path, a scalar quantity
displacement = vector connecting a point to the origin or from one point to another point later in time (in SI, abbreviation for displacement is s)

Instantaneous Velocity versus Average Velocity

- **speed** is rate of change of distance
- **velocity** is rate of change of displacement
- speed is a scalar quantity, velocity is a vector
- direction of velocity vector is same as direction of motion
- **average velocity** (\bar{v}) is the change in displacement over a finite duration, Δt (delta t)
- **instantaneous velocity** (v) is slope (tangent) to the displacement-time curve at a particular instant (t_i)



Differentiation



$$\text{instantaneous velocity} = v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$\text{average velocity} = \bar{v} = \frac{\Delta s}{\Delta t}$$

Example: What is the average velocity of a person who takes 1.200 seconds to cover a distance of 5.00 m?

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{5.00}{1.2} = 4.17 \text{ m/s}$$

Acceleration

- rate of change of velocity
- rate of change of rate of change of displacement
- second derivative of displacement with respect to time
- a vector quantity usually in m/s^2
- in Newtons' notation written as: $a = \dot{v} = \ddot{s}$
- in Leibnitz's notation (standard calculus):

$$\text{instantaneous acceleration} = a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

$$\text{average acceleration} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

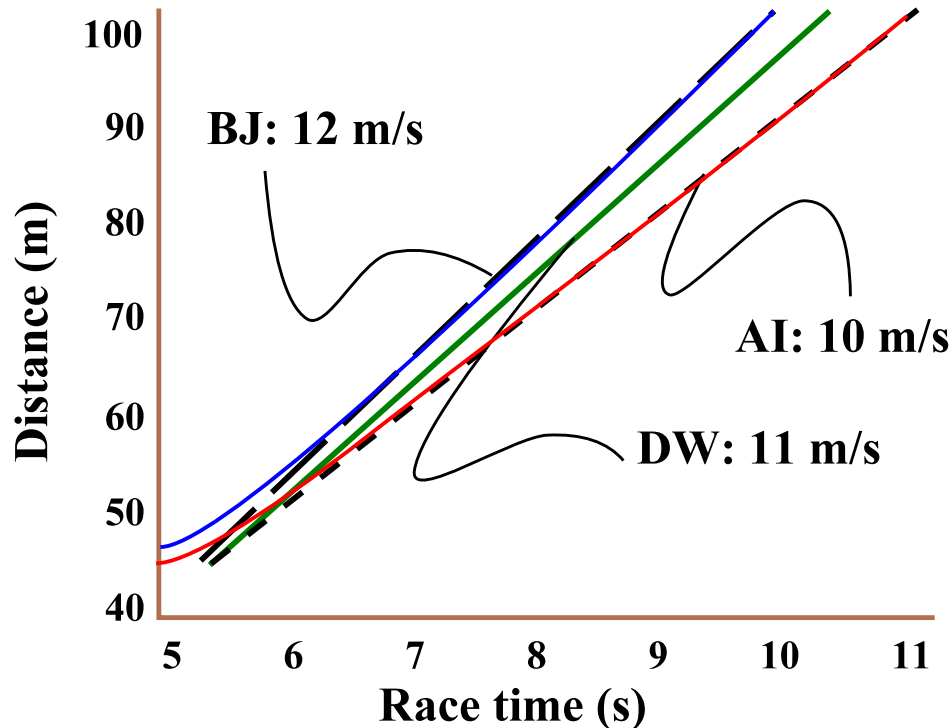
Examples: What is the average acceleration of an object that starts sliding along a tabletop with a velocity of 2.50 m/s and comes to a stop in 3.50 seconds?

$$\bar{a} = \frac{v_f - v_i}{t} = \frac{0 - 2.50}{3.50} = -0.714 \text{ m/s}^2$$

What is the final velocity of a person who accelerates at the rate of 1.500 m/s^2 for 4 seconds from an initial velocity of 2.00 m/s?

$$v_f = v_i + at = 2.00 + 1.500(4) = 8.00 \text{ m/s}$$

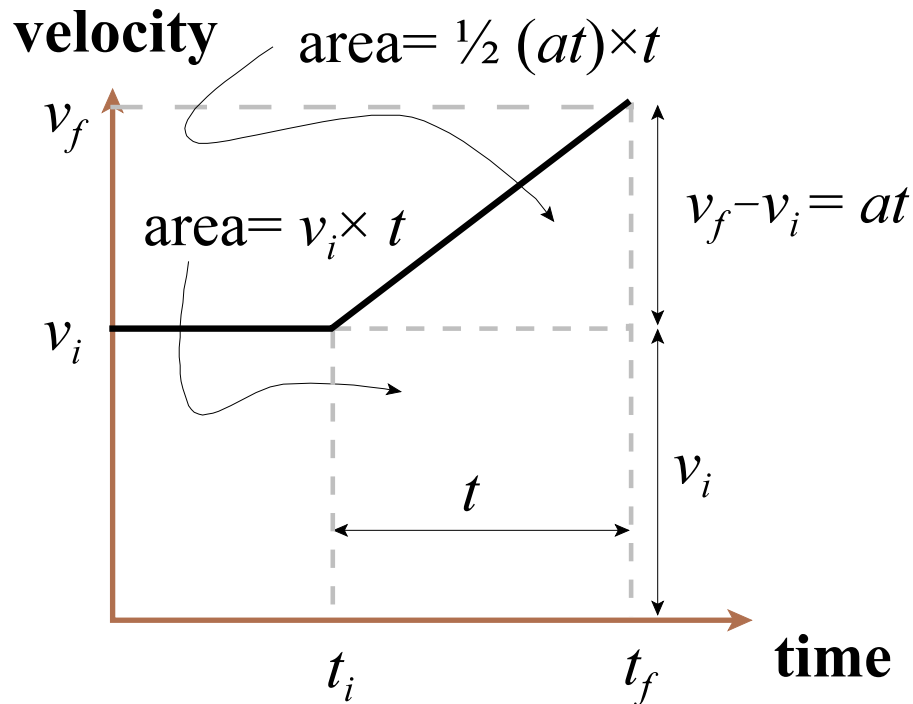
Displacement Histories of Sprinters



- acceleration occurs from start of race until maximum velocity is achieved
- essentially these athletes run last portion of race at their maximum constant velocity
- an athlete may appear to decelerate but may actually be running at a slower velocity

Integration of a Motion with Constant Acceleration

- given a constant velocity of v_i until time t_i
- then a constant acceleration, a , until time t_f
- displacement is area under **velocity** vs. **time** history



$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t} = \frac{0 - 2.5}{3.5}$$

Let $t = t_f - t_i$
and $a = (v_f - v_i) / t$

displacement = area of rectangle + triangle

$$s_{i \rightarrow f} = v_i t + \frac{1}{2} a t^2$$

Constant Linear Acceleration Equations

$$v_f = v_i + a t \quad (\text{I})$$

$$s_f = s_i + v_i t + \frac{1}{2} a t^2 \quad (\text{II})$$

$$v_f^2 = v_i^2 + 2a (s_f - s_i) \quad (\text{III})$$

$$s_f = s_i + \frac{1}{2} (v_i + v_f) t \quad (\text{IV})$$

Where:

s_i = initial position

s_f = final position

v_i = initial velocity

v_f = final velocity

a = constant linear acceleration

t = duration from initial to final positions
(i.e., t)

Equations for Constant Linear Acceleration and Their Relationships

No.	Equation	s_i	v_i	s_f	v_f	t	a
I	$v_f = v_i + at$	X	✓	X	✓	✓	✓
II	$s_f = s_i + v_i t + \frac{1}{2}at^2$	✓	✓	✓	X	✓	✓
III	$v_f^2 = v_i^2 + 2a(s_f - s_i)$	✓	✓	✓	✓	X	✓
IV	$s_f = s_i + \frac{1}{2}(v_i + v_f)t$	✓	✓	✓	✓	✓	X

Examples

How far will a person travel if she accelerates at the rate of 3.00 m/s^2 from an initial velocity of 5.00 m/s for 2.40 seconds?

$$\text{Using } s_f = s_i + v_i t + \frac{1}{2} a t^2$$

$$\begin{aligned}\text{Thus } s_f &= 0 + 5.00 \times 2.40 + \frac{1}{2} (3) 2.40^2 \\ &= 12.00 + 8.64 = 20.64\end{aligned}$$

Therefore she will travel 20.6 metres.

How fast will a cyclist be travelling if he constantly accelerates from 10.00 m/s for 100 metres at the rate of 1.100 m/s^2 ?

$$\text{Using } v_f^2 = v_i^2 + 2 a (s_f - s_i)$$

$$v_f^2 = 10^2 + 2 (1.100) (100 - 0)$$

$$v_f^2 = 100 + 220 = 320$$

$$v_f = \sqrt{320} = 17.888$$

Thus, his final velocity will be 17.89 m/s .