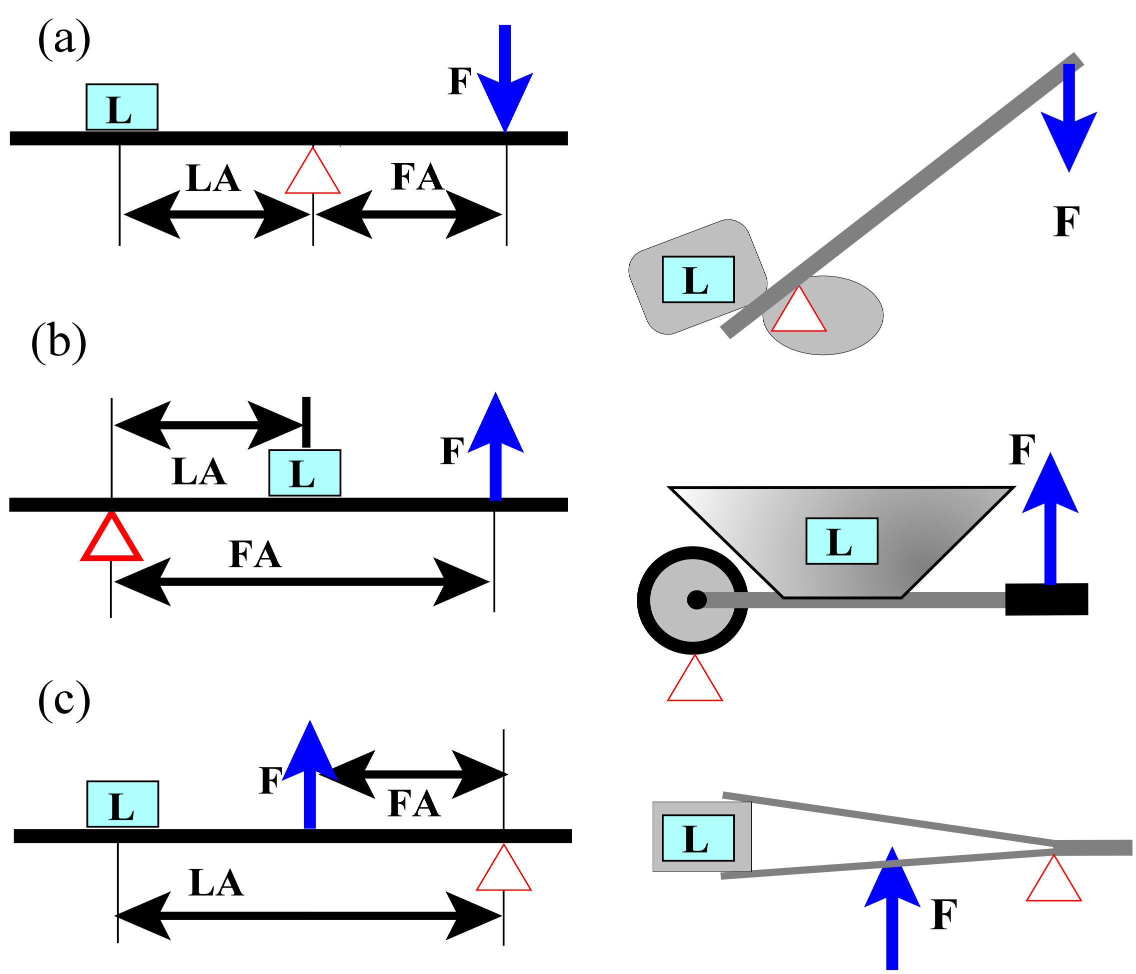
Levers of the Musculoskeletal System

- Lever system consists of:
  - lever
  - fulcrum
  - load
  - force

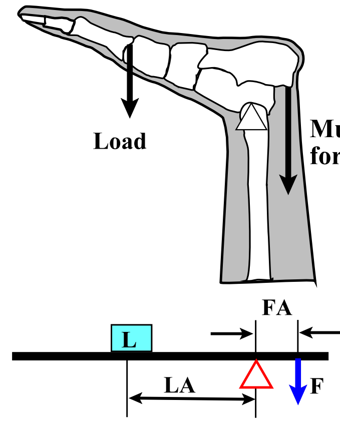
- Three classes of levers
  1. first class (a) - pry bars, crowbars
  2. second class (b) - wheelbarrow
  3. third class (c) - tweezers, most muscles



Levers of the Musculoskeletal System

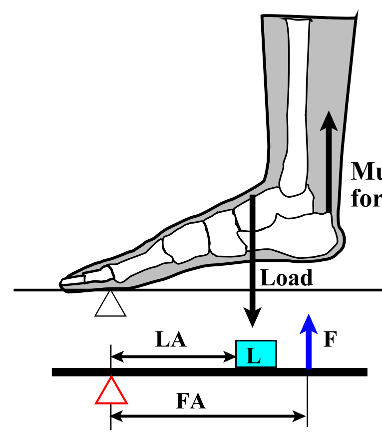
First class lever.

Gastrocnemius inverted



Second class lever.

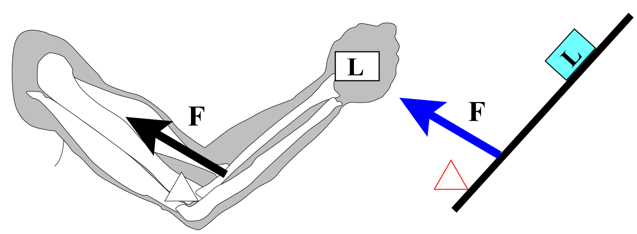
Gastrocnemius
during standing



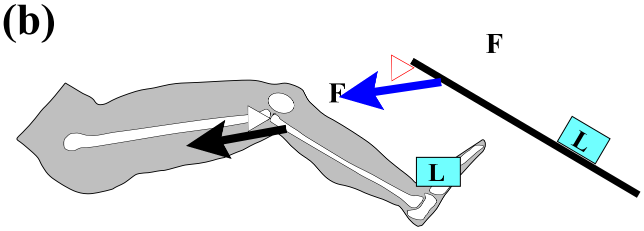
Third class lever.

Biceps brachii and
hamstring muscles

(a)



(b)



Moments of Force

Moment of a Force:

- turning effect of a force
- also called torque (especially when effect is about the longitudinal axis)
- product of moment arm times force

$$M = F\,d \qquad (F \text{ is perpendicular to } d)$$

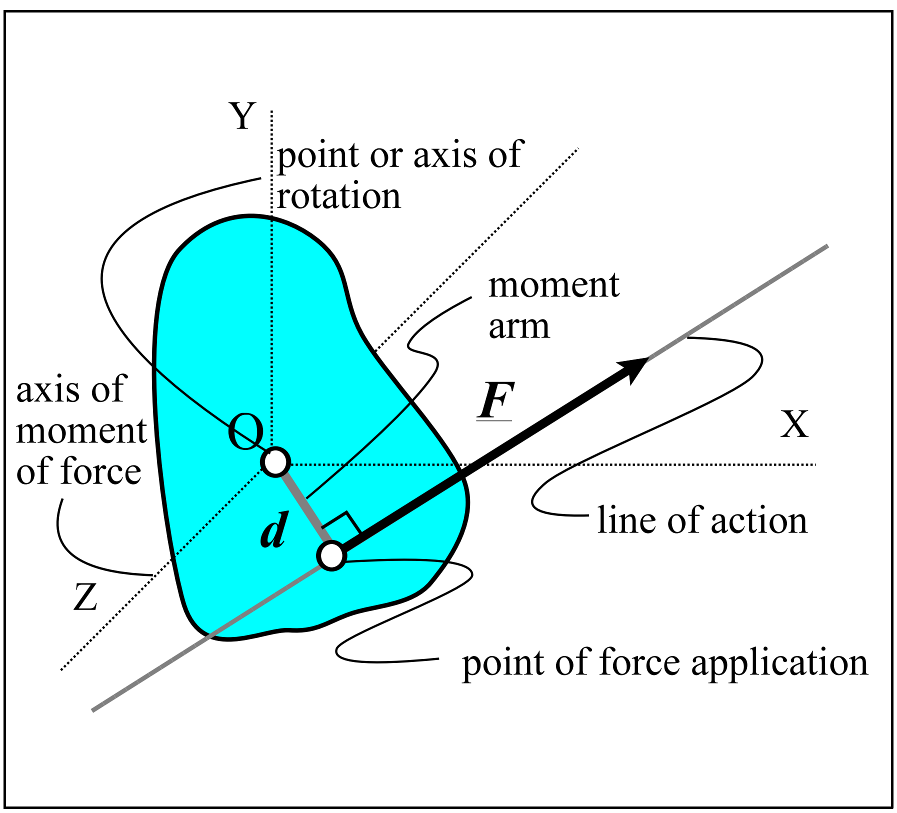
or

$$M = r\,F \sin \qquad (\quad \text{is angle between } F \text{ and } r)$$

- units are newton metres or N.m

Moment: - is the perpendicular distance between a line (e.g., line of force) and a point or an axis. (other uses: moment of inertia, moment of momentum)

Direction: - counter-clockwise (right-hand rule) is positive, clockwise is negative



Moment of a Force

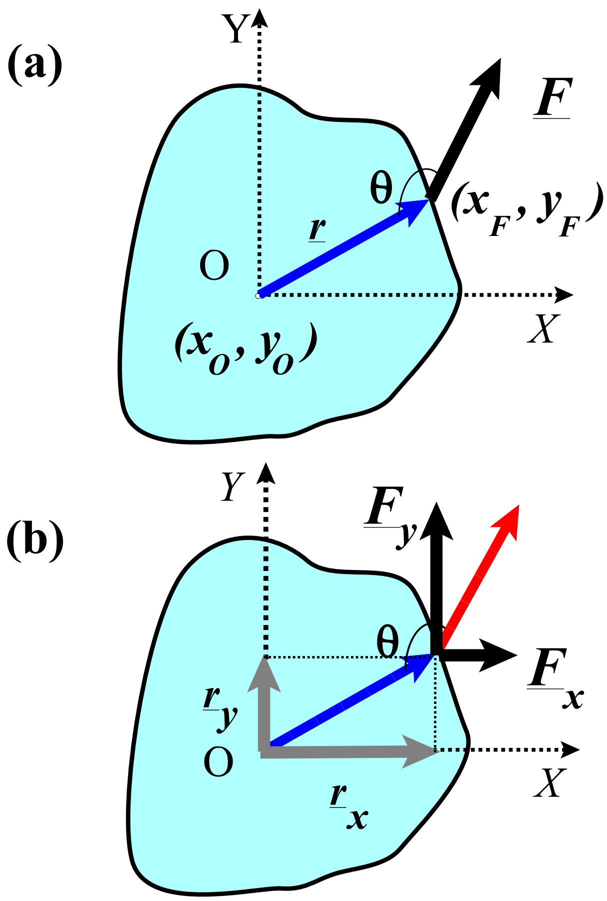
- **vector product** of position vector ($\underline{r}$) and force vector ($\underline{F}$)

$$\underline{\boldsymbol{M}} = \underline{\boldsymbol{r}} \times \underline{\boldsymbol{F}} \qquad \text{(moment vector)}$$

$$= M_z\ \underline{\boldsymbol{k}} \qquad \text{(using unit vector } \underline{k}\text{)}$$

$$= (r_x F_y - r_y F_x)\ \underline{\boldsymbol{k}} \qquad \text{(expansion)}$$

$$[\ \underline{\boldsymbol{M}}\ ]_z = M_z = (r_x F_y - r_y F_x) \qquad \text{(scalar moment about the Z axis)}$$



Example: $\underline{\boldsymbol{r}}$ = (26, 15) cm $\underline{\boldsymbol{F}}$ = (300, 450) N

$$M_z = (26 \times 450) - (15 \times 300) = 7200\ \text{(N.cm)} = 72.0\ \text{(N.m)}$$

Statics

Statics:

- field of applied mechanics concerned with rigid bodies that are in equilibrium

- mechanics of rigid bodies at rest or in constant **linear** motion, also called static equilibrium

- mainly used in the analysis of structures or the stability of bodies (e.g., starting positions)

Laws of Statics

- derived from Newton's First Law:

$$\underline{\boldsymbol{F}} = \underline{\mathbf{0}} \quad \text{or for 2D motions}$$

$$F_x = 0 \text{ and } \quad F_y = 0$$

and

$$(\, \underline{\boldsymbol{r}} \times \underline{\boldsymbol{F}} \,) = \underline{\mathbf{0}} \quad \text{or for 2D motions}$$

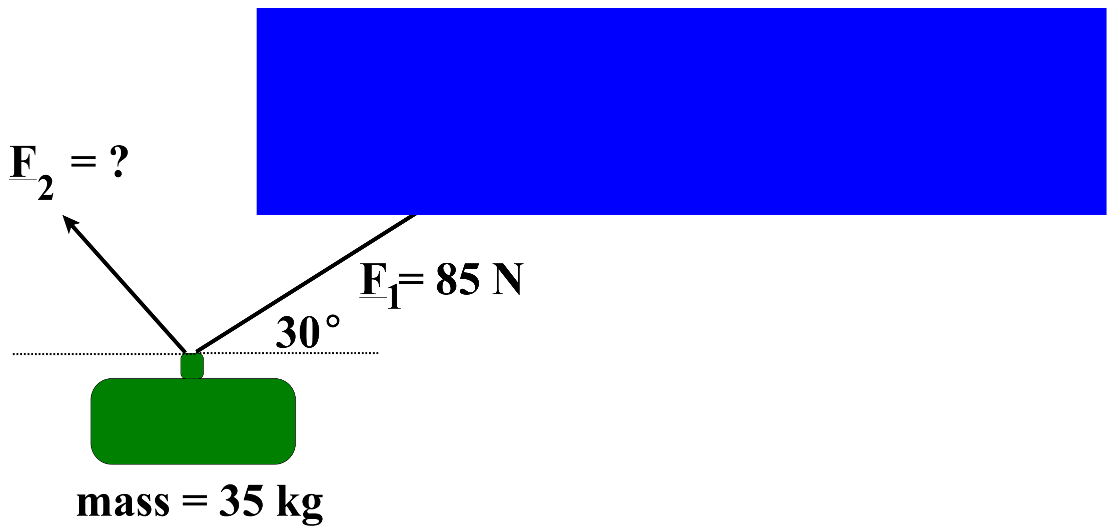
$$M_z = 0$$

- thus, in 2D, there are three independent equations ($F_x = 0$, F_y and $M_z = 0$)

- in 3D there are six equations (3 for forces, 3 for moments)

- i.e., sum of all forces produces a closed polygon and sum of moments equal zero

Example:



$F_x = 0$: $\quad F_{1x} + F_{2x} = 0$

$$85 \cos 30^0 + F_{2x} = 0$$

$$F_{2x} = -\,85 \cos 30^0 = -\,73.6$$

$F_y = 0$: $\quad F_{1y} + F_{2y} - m\,g = 0$

$$85 \sin 30^0 + F_{2y} - (35 \times 9.81) = 0$$

$$F_{2y} = 343 - 85 \sin 30^0 = 301$$

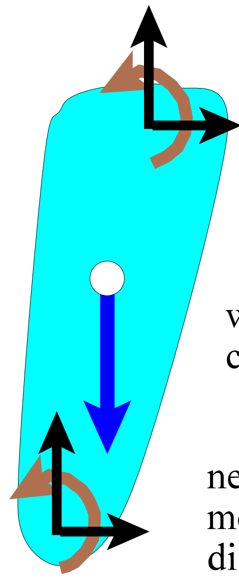
Free-body Diagrams

Rules to construct a valid free-body diagram (FBD).

The following rules must be applied accurately.

Rule 1. **Select** the appropriate body or body segment, keeping in mind the unknowns you want to compute. That is, make sure the free-body includes at least one of the unknowns.

Rule 2. Draw all **known** forces or moments of force at their respective points of application (e.g., weight vector, measured external forces, etc.). Note that the weight vector is placed at the centre of gravity of the free-body.

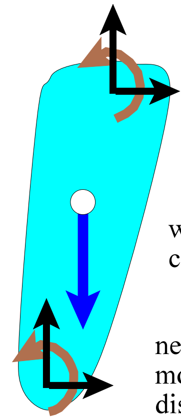


Rule 3. Draw all **unknown** forces and moments that directly contact the free-body. These include points of detachment from other parts of the body and points of contact with the environment. Wherever the free-body is separated from other parts of the body, replace the excluded parts of the body with an unknown force (two components) and a moment of force (see figure below).

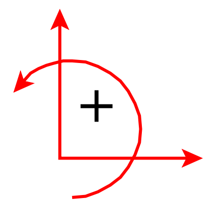
Rule 4. Do NOT include **internal** forces which originate and terminate within the free-body (e.g., muscle or joint forces of internal joints).

Steps for Solving Mechanics Problems

Step 1. Draw a **free-body diagram** (FBD) of the body (or bodies) of interest.



Step 2. Select and draw an appropriate **axis system** that defines the positive directions and slopes of the axes (e.g., X-Y, polar, normal-tangential, radial-transverse, see figure 3.32). Usually a right-handed axis is selected.



Step 3. For each free-body diagram write out the **equations of motion** or **principles** that apply to the problem. These are based upon the six fundamental laws or their corollaries.

$$\text{E.g.,}\quad \Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_z = 0$$

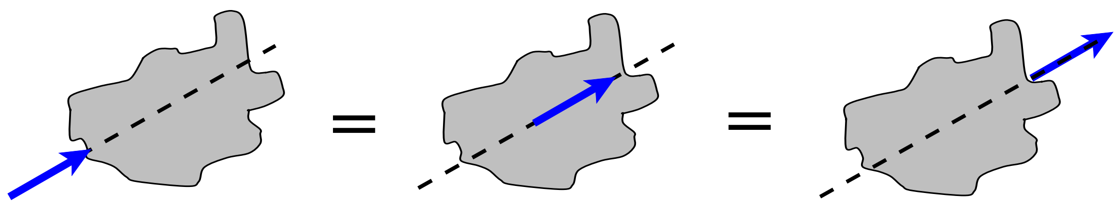
Step 4. **Solve the equations** for the unknowns (Note, this can only be done if the number of unknowns is less than or equal to the number of equations).

Step 5. **Write out the solutions** with the directions and signs (+/–) based on the axes selected in step 2 and with appropriate accuracies and SI units.

Step 6. **Check** your calculations by recomputing the solutions.

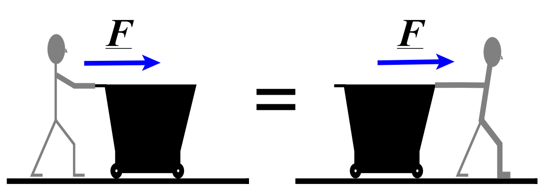
Principle of Transmissibility

One of the six fundamental principles of mechanics. A force acting on a rigid body will produce the same effects no matter where it is applied as long as it is in the same direction and same line of action. We can call such forces, **sliding vectors**, because they can slide up or down their lines of action without affecting their influence on the rigid body.



Force vectors can slide along their lines of action.

push = pull



Deformable bodies do NOT follow transmissibility principle.