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TECHNICAL NOTE

AN ASSESSMENT OF DERIVATIVE DETERMINING TECHNIQUES USED FOR MOTION ANALYSIS*

Abstract—All kinematic studies of human motion employ measurement techniques which introduce noise into displacement data. Commonly, the data, as time related functions, are differentiated to produce velocity and acceleration information. Unfortunately, differentiation amplifies the noise present to such an extent that additional signal treatment is essential.

The study was conducted to compare film generated acceleration curves with the analog acceleration curves of single segment movements. An instrumented segment was manually manipulated by the hand to produce analog records of the angular displacement and acceleration of the segment. Simultaneous filming of the segment produced synchronized displacement data. Acceleration functions were determined from these data using "finite difference," "Chebyshev least squares polynomials" and "digital filtering followed by finite difference" techniques. Digital filtering combined with a first order finite difference technique produced acceleration data very closely approximating the analog signals. The other two techniques were clearly inferior. Data are provided to enable the reader to evaluate his own differentiation procedures.

INTRODUCTION

The accuracy of conclusions drawn from kinematic and kinetic studies of human motion depends largely upon the accuracy of body marker displacement, velocity and acceleration measurements. Calculations of energy, power flow, joint reaction force, torque, centers of rotation, centers of gravity, and other variables are made directly from these marker data. Unfortunately, attempts to calculate velocities and accelerations by successive differentiation of displacement data, such as those produced by film, videotape or digitized analog signals, have been plagued by the amplification of the noise inherent in even apparently smooth displacement curves.

Felkel (1951) evaluated four methods of smoothing and differentiating human motion data and concluded that a grapho-numerical technique was the best at a total estimated error of 20% in the second derivative. However, any graphical procedure is tremendously time consuming and unnecessary with the numerical techniques that have been made practicable with the availability of high speed, large core computers. The simplest numerical method is finite difference but, as pointed out above, amplification of inherent noise produces totally unacceptable under- and over-estimations of the second derivative. The raw data have to be smoothed in some way.

Plagenhoef (1968) has attempted to smooth the displacement data by using a Chebyshev least squares polynomial curve fitting technique as presented by Kuo (1965). Derivatives are then calculated by differentiating the polynomial function. Other least squares techniques could be used, however the Chebyshev method is probably superior in that it remains more stable for higher order polynomials and converges more rapidly on a solution. Recently, Winter *et al.* (1974) has introduced a technique of digitally filtering television displacement data of body segment landmarks. For a given type of filter the sampling rate and upper cutoff frequency is the only information that must be specified by the investigator.

Although a variety of techniques of smoothing biomechanical data have been used by different authors, the only studies in which there was an attempt to evaluate their technique was that by Felkel (1951) and a more recent study by Zernicke *et al.* (1976). The latter investigator compared cubic splines and ordinary polynomials on the basis of force plate output.

The purpose of this paper was to compare acceleration curves produced by an accelerometer with those produced from a synchronized film analysis of the motion of a single segment. The angular acceleration function of the segment was determined from the film data by employing (a) a second order finite difference equation cited by Miller and Nelson (1973), (b) Chebyshev least squares polynomials of degrees 6–16 (whichever produced the minimum error of fit) followed by polynomial differentiation, used by Plagenhoef (1968), and (c) the second order, recursive Butterworth digital filter, used by Winter *et al.* (1974), followed by first order finite difference differentiation (Miller and Nelson, 1973).

METHOD

An aluminium "arm" was fixed to a freely rotating bearing assembly which permitted movement in the horizontal plane (Fig. 1). A Beckman linear potentiometer was placed on the axle to monitor angular displacement while an Endevco piezo-resistive accelerometer was positioned on the free end to measure angular acceleration. A Redlake Locam 16 mm camera, mounted in the ceiling, filmed the motion at 100 frames per sec.

The motion of the instrumented arm was produced by grasping it near the axle and abducting or adducting the hand at the wrist. The displacement and acceleration curves which resulted were within the range of complexity of those found in many human motor tasks and were similar to patterns observed from angular motion of the shank, forearm and other body segments. Indeed, far more complex acceleration curves than those considered here can be produced by the combined actions of several joints during normal and abnormal human motion.

After two preliminary trials, used for calibrating the analog equipment, six trials including simple and complex manual manoeuvres were filmed and the displacement and acceleration signals were synchronously recorded on a Hewlett–Packard instrumentation tape recorder. The start and end of each trial was identified on the analog record-

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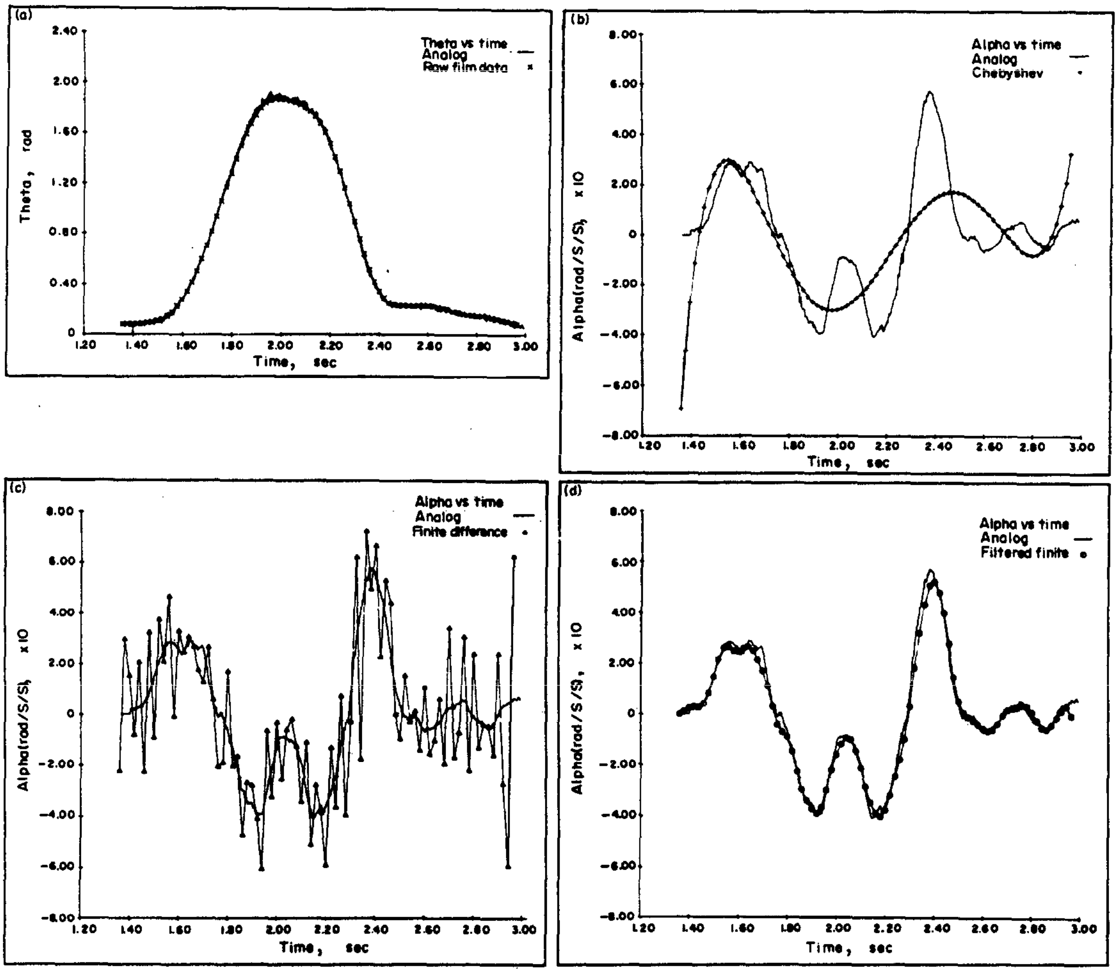

Fig. 2. (a). Raw film and digitized analog angular displacement, (b), (c), (d). Angular acceleration for each differentiation technique compared with accelerometer analog output for simple wrist abduction/adduction movement.

ing by a synch-pulse which also triggered an electronic flash to overexpose one frame on the film. The analog signals were digitized by a PDP 11 minicomputer at 300 samples per sec and written on digital tape. The film was digitized using a CEC digi-table producing x, y coordinates of two points on the segment every 20 msec for the duration of the trial.

RESULTS AND DISCUSSION

Two trials are presented as representative of the results obtained. Figure 2 is a simple abduction–adduction of the segment at moderate range accelerations. Figure 3 demonstrates a more complex movement of 2 abduction-adduction phases, the former executed slowly and the latter executed quite quickly. A cross-section of movement complexity and the resulting accelerations may therefore be observed. Figures 2(a) and 3(a) illustrate the displacement characteristics of each trial showing both the deceivingly smooth raw film data and the unfiltered potentiometer outputs.

Figures 2(b) and 3(b) compare the unfiltered accelerometer outputs with Chebyshev polynomial accelerations determined from 9th and 10th degree fits, respectively, on the displacement data. This technique failed to reproduce the details of the true acceleration function even though the prominent features can be seen in the film generated displacement data. The polynomial fits illustrated were the best of fits up to 16th degree tried.

The results from a second order finite difference equation (Miller and Nelson, 1973) are graphed in Figs. 2(c) and 3(c). The noise content of the raw displacement data has been significantly amplified despite the use of a second order equation. Although the noise produces significant over- and under-estimations, it is apparent that the film data contain the true acceleration information since the "middle" path through the oscillations follows the analog pattern. This suggests that if curve fitting is employed it would be desirable to differentiate the raw data before curve fitting to reduce the noise.

Figures 2(d) and 3(d) demonstrate the influence of digital filtering (Winter *et al.*, 1974) before differentiating the displacement data. The data in these figures were filtered using approx. 7 and 9 Hz cutoff frequencies respectively. The filter phase shift is eliminated by filtering the data forward and then backwards in time. A first order finite difference was used on the filtered data and the resulting acceleration curves showed very little, if any, of the noise apparent in Figs. 2(c) and 3(c). Unquestionably, of the techniques investigated, this method produced the most acceptable estimate of the true acceleration. The minor discrepancies not explainable by the filming techniques can be accounted for by systematic time base shifts (±15 msec) and slight amplitude inaccuracies due to calibration and bias determination of the accelerometer.

A technique not tested in this study is the use of spline function curve fitting. Wold (1974) described the attributes of spline over least squares curve fitting. However, he also

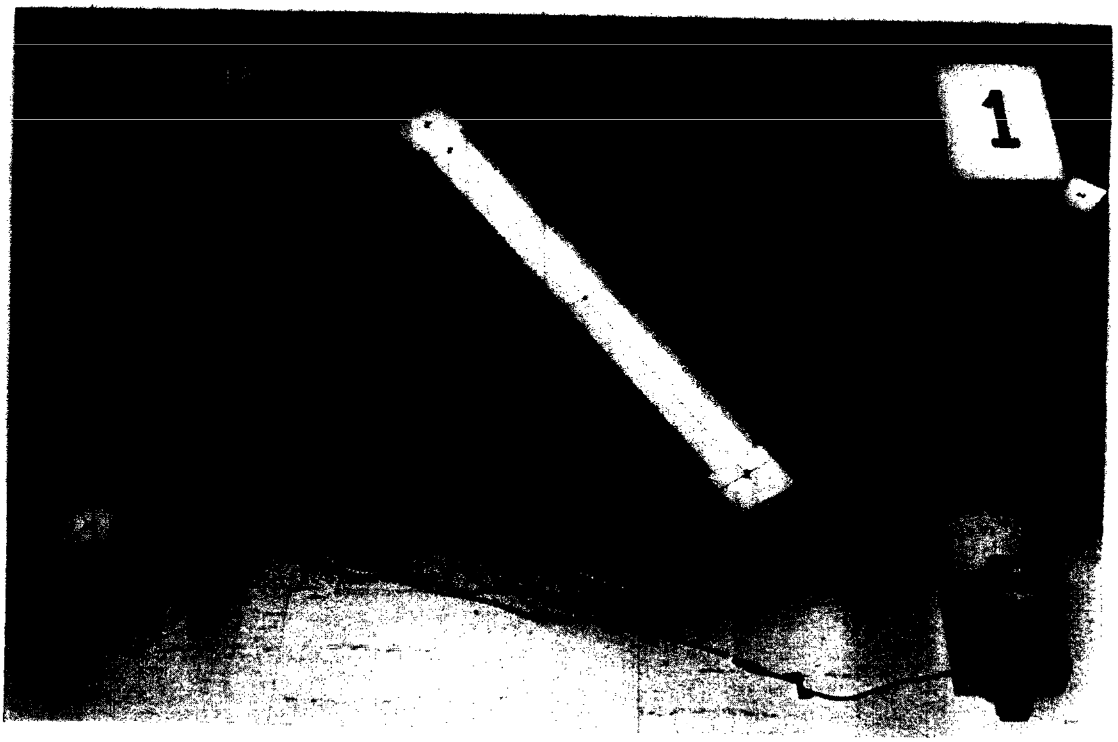

Fig. 1. Mechanical apparatus used for generating analog and film data.

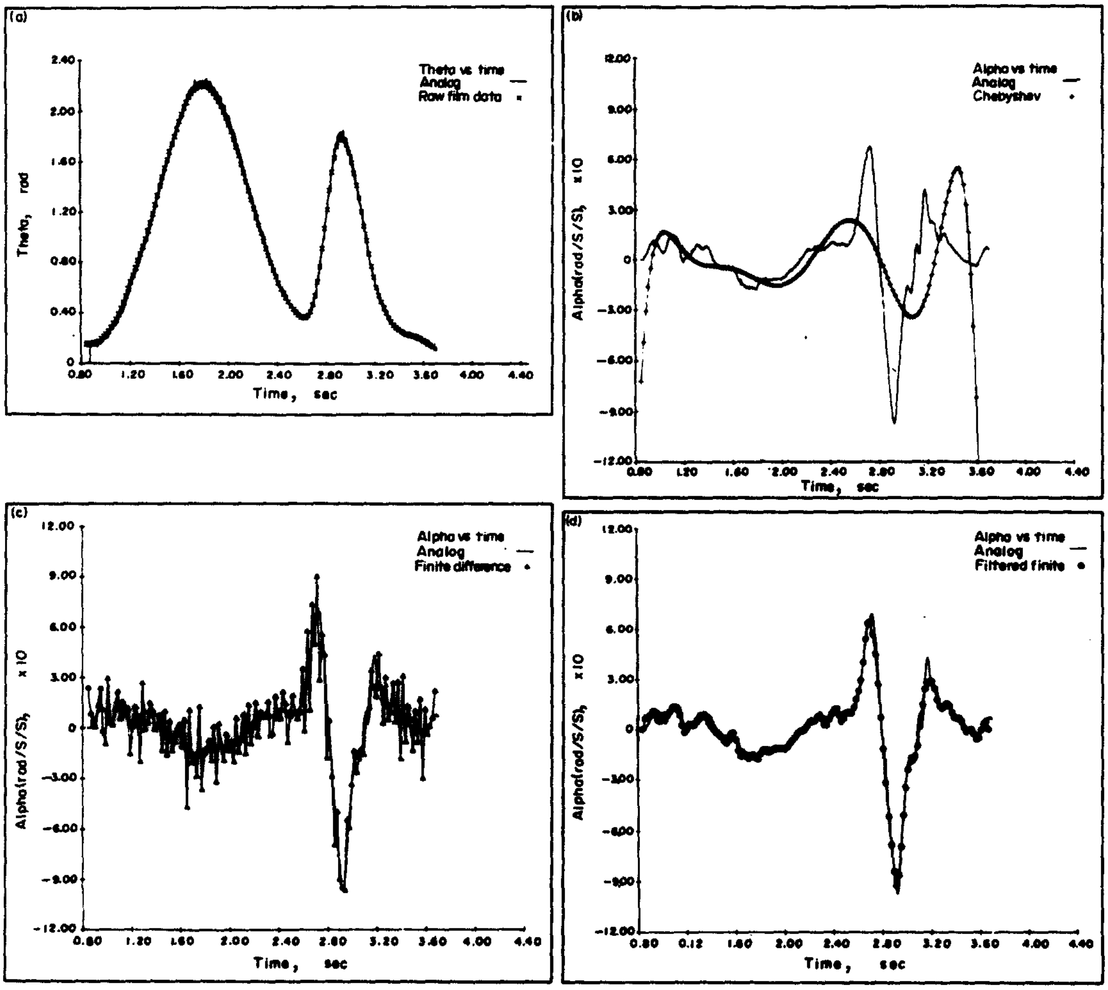

Fig. 3. (a). Raw film and digitized analog displacement. (b), (c), (d). Angular acceleration for each differentiation technique compared with accelerometer digitized analog output for complex wrist abduction/adduction movements.

outlined several conditions concerning the placement of "knots" which necessitates a preliminary analysis to establish their location thereby encumbering the procedure. Moreover, the spline technique employs low order polynomial curve fitting between knots which are to be located at inflection points in the data. Zernicke *et al.* (1976) compared acceleration curves obtained from orthogonal polynomial and cubic spline curve fits of film data with those of the vertical component of force plate generated curves during a kicking movement. He observed an over-smoothing by the orthogonal polynomial and reported that the mean difference between the force platform and film analysis accleration curves was less than 5% for splines but greater than 10% for the orthogonal polynomial.

Our observations of the over-smoothing effect of the Chebyshev polynomial are consistent with Zernicke's report on the orthogonal polynomial technique that he used. Our expectation, however, is that if a spline had been used on the data in Fig. 2(a), the knots for fitting the peak would most likely be located at times 1.75 and 2.25 sec. Looking at the complex acceleration pattern that exists between these points it seems doubtful that spline procedures would capture much more detail in these regions than Chebyshev.

In comparison with Felkel's (1951) data, Chebyshev, and the unfiltered second order finite difference, the digital filtering technique followed by finite difference is clearly superior in terms of the ease and accuracy with which accelerations may be determined. Contrary to the claims of some, accurate knowledge of the frequency spectrum of the signal to be filtered prior to using a digital filter is not necessary. Most human motion will not contain significant harmonics much greater than the 6th observed by Winter *et al.* (1974). Computer time in running the digital filter is relatively small, permitting several runs at different cutoff frequencies. The acceleration curves produced from the finite difference differentiation of filtered data can be compared with the unfiltered differentiated data. The filtered curve should pass through the "middle" path of the unfiltered curve oscillations as demonstrated in Figs. 2(c) and 3(c). If the peaks of the filtered curves are too low, the cutoff frequency of the filter has to be raised. If the oscillations in the filtered curves are too close to those seen in the unfiltered curves, the cutoff frequency must be reduced. Usually two or three runs are sufficient to make these decisions, particularly in applications where a laboratory deals regularly with the same types of movements, for example, walking analysis laboratories. Similar trial and error decisions have to be made in the selection of a polynomial degree or knot locations for splines.

CONCLUSIONS

Digital filtering of the raw film displacement data followed by simple finite difference differentiation was the

Table 1. Raw angular displacement data in rad (read across)

0.1517	0.1492	0.1524	0.1576	0.1635	0.1744
0.1937	0.2126	0.2288	0.2552	0.2833	0.3125
0.3463	0.3879	0.4324	0.4825	0.5358	0.5928
0.6446	0.6966	0.7530	0.8116	0.8639	0.9251
0.9865	1.0494	1.1175	1.1893	1.2610	1.3333
1.4078	1.4778	1.5505	1.6178	1.6855	1.7481
1.8074	1.8658	1.9245	1.9792	2.0339	2.0724
2.1126	2.1452	2.1701	2.1844	2.2012	2.2047
2.2021	2.1940	2.1824	2.1634	2.1433	2.1116
2.0795	2.0427	1.9985	1.9519	1.9000	1.8442
1.7810	1.7189	1.6515	1.5813	1.5132	1.4401
1.3710	1.2987	1.2265	1.1594	1.0909	1.0255
0.9630	0.9033	0.8488	0.7928	0.7359	0.6853
0.6370	0.5913	0.5533	0.5207	0.4856	0.4531
0.4276	0.4055	0.3856	0.3698	0.3665	0.3648
0.3835	0.4091	0.4622	0.5367	0.6447	0.7663
0.9091	1.0680	1.2216	1.3753	1.5169	1.6317
1.7246	1.7819	1.8011	1.7825	1.7405	1.6750
1.5954	1.5090	1.4121	1.3088	1.2000	1.0853
0.9715	0.8626	0.7664	0.6798	0.6014	0.5392
0.4865	0.4359	0.3960	0.3587	0.3254	0.3015
0.2800	0.2676	0.2499	0.2421	0.2323	0.2267
0.2226	0.2140	0.2080	0.1993	0.1953	0.1811
0.1695	0.1565	0.1438	0.1356		

only one of the three techniques studied in these experiments that accurately reproduced the acceleration time curves recorded from an accelerometer. Polynomial curve fitting (of up to 16th degree) smoothed the acceleration curves too much and 2nd order finite difference differentiation did not smooth them enough. The polynomial fit severely attenuated the peaks and distorted the time histories of the true accelerations of the single segment motion investigated. Use of these acceleration data in subsequent calculations of joint torques and joint reaction forces would produce virtually meaningless results. In fact, from the data presented, one would suspect that finite difference differentiation followed by curve fitting would produce more acceptable results since the general shape of the acceleration function is preserved with unfiltered finite difference techniques. Furthermore, close inspection of the digitized analog signal in Figs. 2(a) and 3(a) reveals noise which, if differentiated, would be magnified probably intolerably. Digital filtering of even this signal, prior to differentiation, is therefore recommended.

Since it is impossible to exhaustively assess all techniques being used for determining derivatives, in human motion studies, the reader is invited to assess his own method using the data presented in Table 1 which is identical to the raw angular displacement data (in rad) used in Fig. 3(a) for 142 frames at 0.0201 sec between frames.

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