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Technical note

Design and responses of Butterworth and critically damped digital filters

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Abstract

For many years the Butterworth lowpass filter has been used to smooth many kinds of biomechanical data, despite the fact that it is underdamped and therefore overshoots and/or undershoots data during rapid transitions. A comparison of the conventional Butterworth filter with a critically damped filter shows that the critically damped filter not only removes the undershooting and overshooting, but has a superior rise time during rapid transitions. While analog filters always create phase distortion, both the critically damped and Butterworth filters can be modified to become zero-lag filters when the data are processed in both the forward and reverse directions. In such cases little improvement is realized by applying multiple passes. The Butterworth filter has superior 'roll-off' (attenuation of noise above the cutoff frequency) than the critically damped filter, but by increasing the number of passes of the critically damped filter the same 'roll-off' can be achieved. In summary, the critically damped filter was shown to have superior performance in the time domain than the Butterworth filter, but for data that need to be double differentiated (e.g. displacement data) the Butterworth filter may still be the better choice.

Keywords: Data smoothing; Digital filtering; Signal processing; Kinematics

1. Introduction

Lowpass digital filtering of noisy signals has for many years been an essential procedure for biomechanists. For a comprehensive theory of the design of these filters, the reader is referred to chapter 13 of Valkenburg [1]. Probably, the most widely used filtering method in human movement analyses was first published by Winter et al. [2] and was later shown by Pezzack et al. [3] to successfully reduce the noise in kinematic signals and their derivatives. The filter was an underdamped, Butterworth, zero-lag filter which was transformed into a critically damped filter by Dowling [4]. Butterworth filters are often chosen for smoothing movement data because they are optimally flat in their pass-band, have relatively high roll-offs and rapid response in the time domain. A method of calculating the filtering coefficients for both filters was reported by Winter [5], but there was an error in the cutoff frequency determination for multiple passes of these filters and in the rise time comparison between the two filters. The purposes of this investigation were to present the correct formulas for the determination of the filtering coefficients and to compare the time and frequency domain responses of these two filters on a known set of data.

2. Theory

A step function of unity amplitude was created to compare the response characteristics of the filters. These data were filtered in various ways by both a critically damped and a Butterworth filter. These ways include changing the order of the filter, using zero-lag and non-zero-lag verisons of the filters and changing the cutoff frequencies. To approximate an infinite impulse response analog filter in the digital domain, the bilinear transform

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[6] is used for a one-to-one continuous mapping from the s-plane to the z-plane. This causes a frequency dependent 'warping' of the desired cutoff frequency which is corrected with the following equation [7]. To make a zero-lag filter, the data were passed through the filter twice (once in the forward direction and once in reverse). To maintain the correct cutoff when using multiple passes of a filter (cascading) the cutoff frequencies must be adjusted. The following equations adjust the critically damped and Butterworth filters, respectively.

$$c_{Bw} = \frac{1}{\sqrt[4]{2^{\frac{1}{n}}-1}} \quad (1)$$

$$f^*_{Bw} = f_{Bw} \times c_{Bw}$$

$$c_{crit} = \frac{1}{\sqrt{2^{\frac{1}{2\cdot n}}-1}} \quad (2)$$

$$f^*_{crit} = f_{crit} \times c_{crit}$$

where, f_{crit} and f_{Bw}, are the desired cutoff frequencies of the critically damped and Butterworth filters, respectively, f^*_{crit} and f^*_{Bw}, are the adjusted cutoff frequencies necessary to produce the requested cutoff and n, is the number of filter passes. Notice that the Butterworth filter's cutoff is not adjusted when the data are passed once ($n = 1$), but the critically damped filter must be adjusted even for a single pass (this was not reported by Winter [5]).

The next step is to determine the corrected angular cutoff frequency of ω^*_c the lowpass filter (see Winter [8] or Murphy and Robertson [9]) where f_{sr} is the sampling rate in hertz.

$$\omega^*_c = \tan\left(\frac{\pi f^*_{crit}}{f_{sr}}\right) \text{ or } \omega^*_c = \tan\left(\frac{\pi f^*_{Bw}}{f_{sr}}\right) \quad (3)$$

To compute the critically damped lowpass filter coefficients let $K_1 = \sqrt{2}\omega^*_c$ for the Butterworth filter (cf. Winter [5]) or $K_1 = 2\omega^*_c$ for the critically damped filter and for both filters let $K_2 = (\omega^*_c)^2$. The lowpass coefficients become:

$$a_0 = a_2 = \frac{K_2}{1 + K_1 + K_2}; \ a_1 = 2a_0; \quad (4)$$

$$b_1 = 2a_0\left(\frac{1}{K_2}-1\right); \ b_2 = 1-(a_0 + a_1 + a_2 + b_1) \quad (5)$$

The following equation is the second-order recursive (infinite impulse response, IIR) filter:

$$y_n = a_0x_n + a_1x_{n-1} + a_2x_{n-2} + b_1y_{n-1} + b_2y_{n-2} \quad (6)$$

or preferably

$$y_n = a_0(x_n + 2x_{n-1} + x_{n-2}) + b_1y_{n-1} + b_2y_{n-2} \quad (7)$$

3. Results and discussion

Fig. 1 shows the original step function compared with the output from a critically damped and a Butterworth (underdamped) filter (both of which are 4th order, zero-lag). Sampling rate and cutoff frequency for both filters were set at 100 and 10 Hz, respectively. Notice that, as expected, the critically damped filter does not undershoot or overshoot the data. The Butterworth filter produces an incorrect maximum that is approximately 5% too high. Furthermore, the Butterworth filter takes longer to return to unity after the step by approximately 0.04 s. This is contrary to what would be expected from analog filters and to what was reported by Winter [5] (p. 41). He stated 'Critically damped filters . . . suffer from a slower rise time'. Clearly it is the Butterworth filter that has the slower rise time and also the slower 'fall time'.

If the step function represented a golf ball that was struck, the undershoot of the Butterworth filter immediately before the step or club contact would mean that the ball started moving in the opposite direction before the club head even touched the ball. Of course, the critically damped filter is not perfect; the filtered data would show the ball moving before contact, but with a smaller lead time and at least moving in the correct direction compared with the Butterworth filter.

Fig. 2 compares the step function with both a zero-lag and a nonzero-lag filter. The nonzero-lag filter significantly lags behind the original data. Note that the zero-lag filter responds to the step before the rise occurs; it appears to 'anticipate' the step. The nonzero-lag filter responds only after the step has been encountered, which inevitably produces a time lag in comparison to the original signal. Despite the slight anticipation produced by the zero-lag filter, its elimination of lag makes it clearly superior to the nonzero-lag filter for most purposes.

Fig. 3 shows the effects produced by changing the cutoff frequency of a zero-lag, critically damped filter. Cutoff frequencies of 1, 2, 4 and 10 Hz were applied.

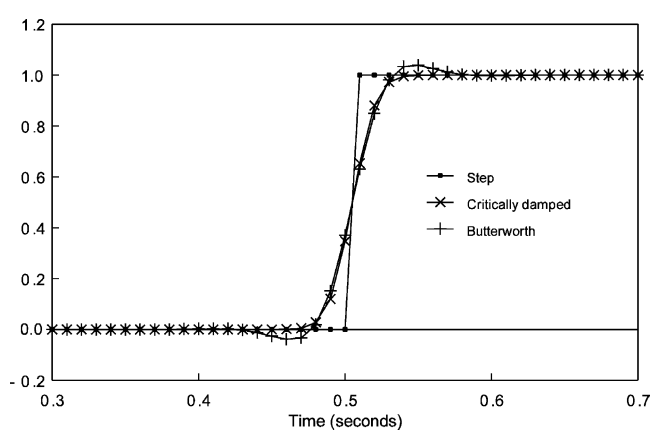

Fig. 1. Comparison of the responses of the critically damped and Butterworth filters to a step function.

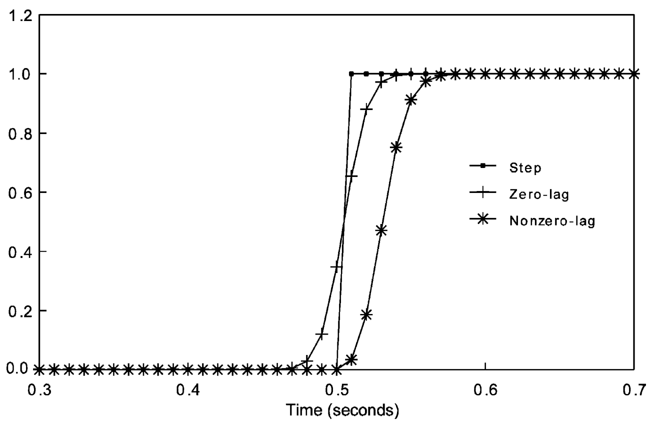

Fig. 2. Comparison of the responses a zero-lag and nonzero-lag critically damped filter to a step function.

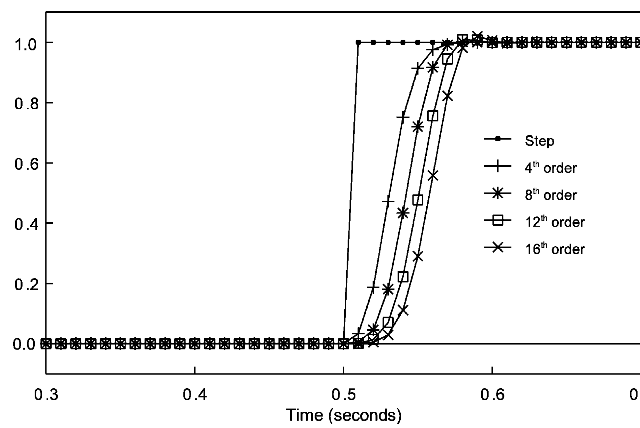

Fig. 4. Effects of cascading a nonzero-lag critically damped filter.

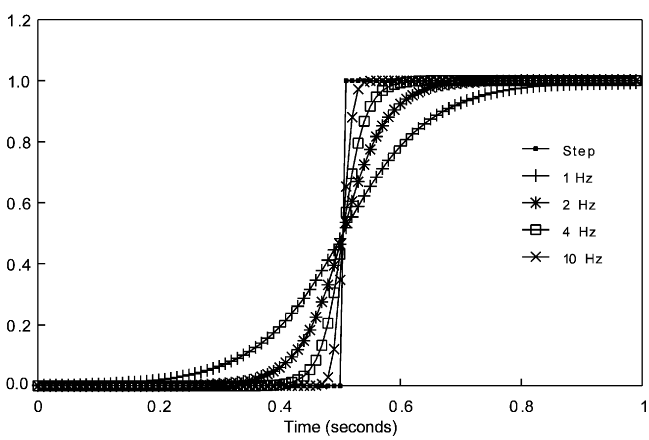

Fig. 3. Effects of varying the cutoff frequency of a critically damped filter.

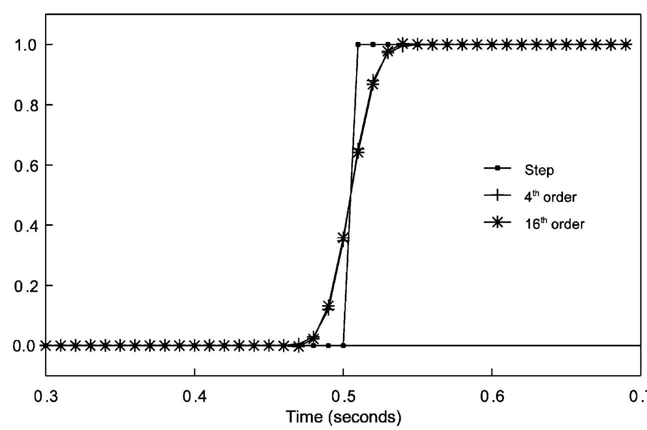

Fig. 5. Effects of cascading a zero-lag critically damped filter.

As expected, varying the cutoff frequency significantly affected the ability of the filter to follow the original step function. The poorest match occurs with the 1 Hz cutoff frequency. It should be noted, however, that while producing a better reproduction of the step function, a higher cutoff will be unable to eliminate low frequency noise from the original data. This is problematic whenever first and especially second time derivatives are taken (cf., Winter [5]).

Fig. 4 shows the effects of 'cascading' a 2nd order, nonzero-lag, critically damped filter. This increases the rise time and the order of the filter. After adjusting the cutoff frequency for the number of repetitions (Eq. (2)), the data were filtered 2, 4, 6 and 8 times with the same filter. This produced the equivalent of 4th, 8th, 12th and 16th order filters. In all cases a sampling rate 100 Hz and a cutoff frequency of 10 Hz were used. Thus, the lowest order filter had the least phase lag while the 16th order filter had the greatest lag.

Fig. 5 shows the results of cascading a critically damped, zero-lag filter one and four times, producing 4th and 16th order filters. The equivalent 8th and 12th order filters were not plotted since their results were virtually identical to the others. Note that a zero-lag filter, passes over the data twice—once in the forward (time) direction and once in reverse—increasing the order of the filter by four. In Fig. 5, there is apparently no difference between the different orders. These results suggest that the order of the filter is irrelevant to the time domain response when using a zero-lag filter. That is, the fourth-order filter produces almost identical time domain results to that of the 16th order, but with reduced computation and time. In this example, the cutoffs were all adjusted to achieve the same cutoff frequency so there is little difference in the rise times. Note that results for the Butterworth filter are not shown but the same responses occurred. That is, increasing the order of a zero-lag Butterworth filter by increasing the number of passes did not significantly affect the rise times.

The attenuation of frequency components above the

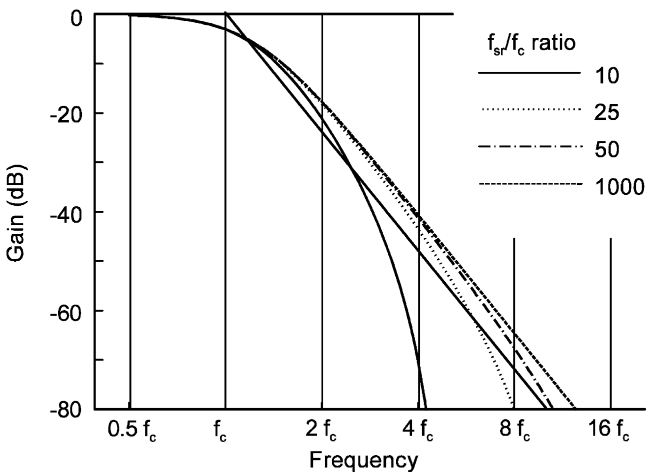

Fig. 6. Comparison of roll-offs of critically damped filter with different f_{sr}/f_c ratios. The darker straight line shows the roll-off for an ideal 4th order filter.

cutoff frequency is often referred to as 'roll-off'. Both the Butterworth and critically damped filters used in this study are second-order filters that should theoretically have an attenuation of 12 dB per octave. A dual pass through these filters would increase the order to four and should result in an attenuation of 24 dB per octave. The actual roll-off is a function of the ratio of sampling rate (f_{sr}) and the chosen cutoff frequency (f_c). Fig. 6 shows the actual roll-off of different f_{sr}/f_c ratios of a single dual pass of the Butterworth filter along with the ideal response (thick straight line). It can be seen that the sharpness of the cutoff is affected by this ratio and is not the same as the theoretical response.

The critically damped filter not only has a different response in the time domain as shown earlier, but the roll-off is also different from that of the Butterworth filter. Fig. 7 shows that the roll-off of the critically damped filter is not as sharp as that of the Butterworth filter and it requires about five dual passes to achieve similar attenuation above the cutoff frequency. Although this response is shown for a f_{sr}/f_c ratio of 100, the five dual passes are needed for all ratios for the critically damped filter to have a similar response to the single dual pass of the Butterworth filter. It should be noted that the number of dual passes is only a minor consideration in terms of computation time with the speed of today's computers and that even low order filters achieve dramatic attenuations of high frequency noise. A gain of −40 dB is an attenuation of 99% of the noise and an examination of Figs. 6 and 7 shows that this is often achieved within the first two octaves. Dramatically increasing the order of the filter by using many passes has diminishing returns in terms of noise reduction and quite often the precision limit of the computer prevents further noise reduction.

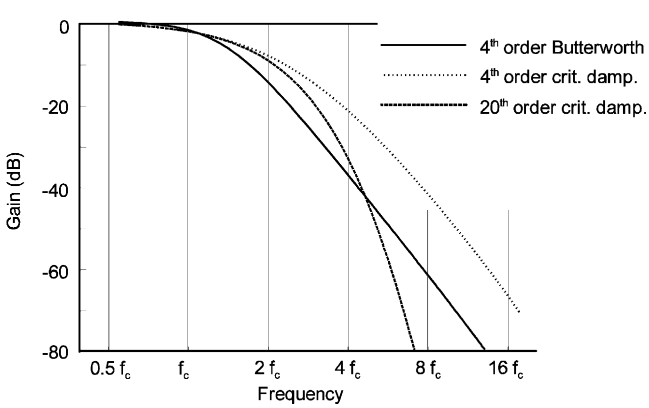

Fig. 7. Comparison of roll-offs for a 4th order Butterworth filter and a 4th and 20th order critically damped filter.

In summary, the critically damped filter is preferable to the Butterworth for signals with rapid transitions and particularly if these signals do not require time differentiation, for example, force, accelerometer and EMG signals. The critically damped filter will produce more accurate estimates of the temporal occurrence of transitions and will produce no over- or under-shooting of the original data.

When filtering signals that will be differentiated or double-differentiated, such as displacement signals, the critically damped filter is weaker at removing noise above the cutoff frequency. This is because the amplitude of high frequency noise increases after differentiation [5]. However, by increasing the number of passes (usually dual passes) and hence its order, a suitable roll-off equivalent to the Butterworth can be achieved. We recommended five dual passes (20th order) to match the Butterworth's single dual pass (4th order). No additional filter passes are needed if no time derivatives (and especially double differentiation) are to be taken.

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