

Measurement of Internal Work

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Mechanical Energy

- Four forms
 - Gravitational potential $m g y$
 - Elastic potential $\frac{1}{2} k s^2$
 - Translational kinetic $\frac{1}{2} m v^2$
 - Rotational kinetic $\frac{1}{2} I \omega^2$
- Total mechanical energy is sum of all four
- Elastic potential energy is usually omitted because it cannot be measured accurately

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Total Body Mechanical Energy

- Sum of all segmental total mechanical energies (E_s)
- Segmental total energy
$$E_s = m_s g y_s + \frac{1}{2} m_s v_s^2 + \frac{1}{2} I_s \omega_s^2$$
- Total body energy

$$E_{\text{total}} = \sum E_s$$

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External Work

- Mechanical work = change in mechanical energy

$$W_{\text{ext}} = \sum (\Delta E_{\text{total}})$$

- or simplified

$$W_{\text{ext}} = E_{\text{total}}(t_n) - E_{\text{total}}(t_0)$$

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Zero-work Paradox

- If a body moves at constant velocity along a horizontal path no “external” mechanical work is done!
- Mechanical cost of moving body parts cancel out if motion is cyclic.
- Problem exists with all locomotor tasks but if body speeds up or rises, some external work is done, however, additional costs of “cycling” the body parts are not included.

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Work Allowing No Transforms or Transfers of Energy

- Work done that prevents changes of forms of energy, i.e., from potential to kinetic, etc.

$$W_n = \sum \sum | \Delta m_s g y_s | + | \Delta \frac{1}{2} m_s v_s^2 | + | \Delta \frac{1}{2} I_s \omega_s^2 |$$

- First summation is over all time intervals
- Second summation is over all segments
- Norman et al. (1976)

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Work Allowing within Segment Transforms

- Work done that permits changes of forms of energy within a segment (kinetic to potential and *vice versa*) but no transfers from segment to segment

$$W_w = \sum \sum | \Delta E_s |$$

- First summation is over all time intervals
- Second summation is over all segments
- Winter (1979)

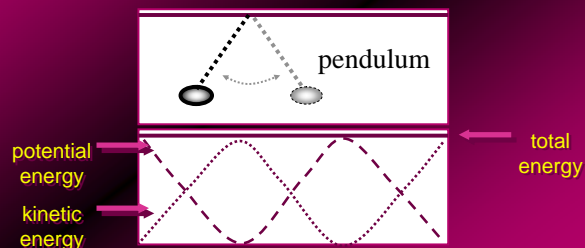
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Energy Conservation by Transforming Energy within a Segment

- simple pendulum



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Internal Work

- Internal work measures the mechanical costs of moving the limbs during a cyclic motion. The equation permits transfers of energy from segment to segment and from one form to another.

$$W_{\text{int}} = \sum |\Delta E_{\text{total}}| - W_{\text{ext}}$$

- Absolute values prevent decreases in mechanical energy from canceling increases

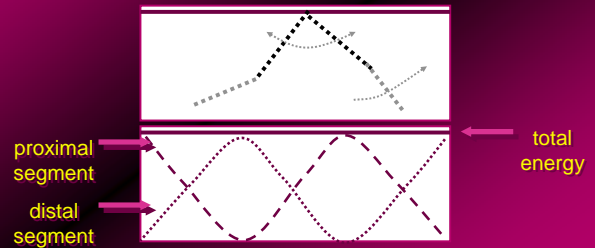
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Energy Conservation by Transferring Energy between Segments

- compound pendulum



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Energy Saved by Transfers and Transforms

- Energy saved by permitting transfers of energy from segment to segment

$$E_{\text{transfers}} = W_w - W_{\text{int}}$$

- Energy saved by permitting transforms of energy from one form to another (potential of kinetic)

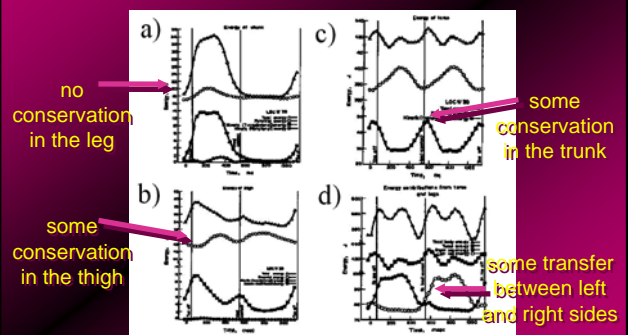
$$E_{\text{transforms}} = W_n - W_w$$

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Segment Energies during Walking



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