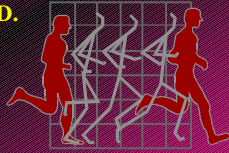


Mechanical Energy, Work and Power

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Energy

- Ability to do work
- Measured in joules (J)
- One joule is the work done when a one newton force moves an object through one metre
- 1 Calorie = 1000 cal = 4.186 kJ
- Can take many forms

Forms of Energy

- Mass ($E = mc^2$)
- Solar or Light (solar panels, photovoltaic battery)
- Electricity (electron flux, magnetic induction)
- Chemical (fossil fuels, ATP, food)
- Thermal or Heat
- Mechanical energy

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Types of Mechanical Energy

- Translational Kinetic = $\frac{1}{2} m v^2$
 - $v^2 = v_x^2 + v_y^2 (+ v_z^2)$
 - this is usually the largest type in biomechanics
- Rotational Kinetic = $\frac{1}{2} I \omega^2$
 - this is usually the smallest type in biomechanics
- Gravitational Potential = $m g y$
- Elastic Potential = $\frac{1}{2} k s^2$
 - Assumed to be zero for rigid bodies

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Laws of Thermodynamics

- Zeroth law
 - When two quantities are in thermal balance to a third they are in thermal balance with each other. I.e., they have the same temperature.
- First Law (Law of Conservation of Energy)
 - Energy is conserved (remains constant) within a "closed system."
 - Energy cannot be created or destroyed.
- Second Law (Law of Entropy)
 - When energy is transformed from one form to another there is always a loss of usable energy.
 - All processes increase the entropy of the universe.
- Third Law
 - Absolute zero (absence of all atomic motion) cannot be achieved.

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Law of Conservation of Mechanical Energy

- If the resultant force acting on a body is a conservative force then the body's total mechanical energy will be conserved.
- Resultant force will be conservative if all external forces are conservative.
- A force is conservative if it does no work around a closed path (motion cycle).

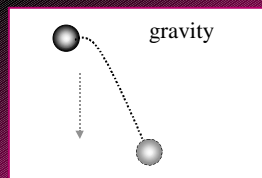
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Examples of Conservative Forces

- Gravitational forces



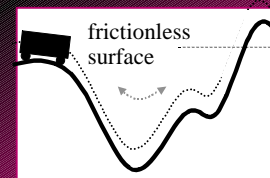
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Examples of Conservative Forces

- Gravitational forces
- Normal force of a frictionless surface



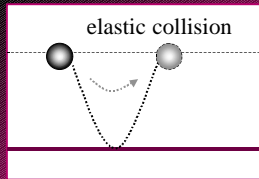
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Examples of Conservative Forces

- Gravitational forces
- Normal force of a frictionless surface
- Elastic collisions



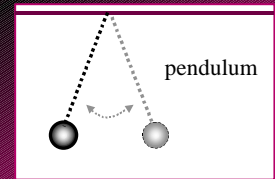
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Examples of Conservative Forces

- Gravitational forces
- Normal force of a frictionless surface
- Elastic collisions
- Pendulum



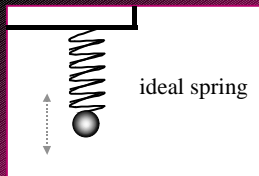
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Examples of Conservative Forces

- Gravitational forces
- Normal force of a frictionless surface
- Elastic collisions
- Pendulum
- Ideal spring



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Direct Ergometry

- Treadmill Ergometry
- External work = $m g t v \sin \theta$
- where, m = mass, $g = 9.81$, t = time, v = treadmill velocity, and θ = treadmill's angle of incline



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Direct Ergometry

- Cycle Ergometry
- External work = $6 n L g$
- where, n = number of pedal revolutions, L = load in kiloponds (kp) and $g = 9.81$
- $1 \text{ kp} = 9.81 \text{ newtons}$
- Note, each pedal cycle is 6 metres motion of flywheel



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Direct Ergometry

- Gjessing Rowing Ergometry
- External work = $n L g$
- where, n = number of flywheel cycles, L = workload in kiloponds and $g = 9.81$



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Biomechanical Methods

- Point Mass Method
 - Simplest, least accurate, ignores rotational energy
- Mechanical energy = $E = m g y + \frac{1}{2} m v^2$
- External work = $E_{\text{final}} - E_{\text{initial}}$



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Biomechanical Methods

- Single Rigid Body Method
 - Simple, usually planar, includes rotational energy
- Mechanical energy = $E = mgy + \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$
- External work = $E_{\text{final}} - E_{\text{initial}}$



Carriage load

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Biomechanical Methods

- Multiple Rigid Body Method
 - Difficult, usually planar, more accurate, accuracy increases with number of segments
- External work = $E_{\text{final}} - E_{\text{initial}}$
- E = sum of segmental total energies (kinetic plus potential energies)



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Biomechanical Methods

- Inverse Dynamics Method
 - Most difficult, usually planar, requires force platforms
- External work = $\Sigma (\Sigma M_j \omega_j \Delta t)$
- Sum over all joint moments and over duration of movement



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Work of a Force

Work of a force is product of force (F) and displacement (s) when F and s are in the same direction.

$$\begin{aligned} \text{Work} &= F s && \text{(when F is parallel to s)} \\ &= F s \cos \phi && \text{(when F is not parallel to s} \\ &&& \text{and is } \phi \text{ angle between F and s)} \\ &= \mathbf{F} \cdot \mathbf{s} = F_x s_x + F_y s_y && \text{(dot or scalar product)} \\ &= E_f - E_i && \text{(change of energy)} \\ &= P t && \text{(power times time)} \end{aligned}$$

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Work of a Moment of Force

Work of a moment of force is product of moment of force (M) and angular displacement (θ).

$$\begin{aligned} \text{Work} &= M \theta \\ &= r F (\sin \phi) \theta && \text{(\phi is angle between r and F)} \\ &= P t && \text{(mean power times duration)} \\ &= \Sigma (M \omega \Delta t) && \text{(time integral of moment power)} \end{aligned}$$

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Average Power

Power is the rate of doing work.
 measured in watts (W), 1 watt = 1 joule per second (J/s)

Power = work / time (work rate)
 = $(E_f - E_i) / \text{time}$ (change in energy over time)
 = $(F s) / t = F v$ (force times velocity)
 = $(M \theta) / t = M \omega$ (moment of force times angular velocity)

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Instantaneous Power of a Force or Moment of Force

Power = $F v$ (when F is parallel to v)
 = $F v \cos \phi$ (when F is not parallel to v and is ϕ angle between F and v)
 = $\underline{F} \cdot \underline{v} = F_x v_x + F_y v_y$ (dot or scalar product)
 = $M \omega$ (moment times angular velocity)

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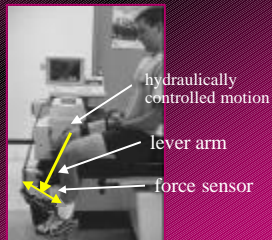
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Isokinetic Dynamometers

KinCom 500H

- Controls speed of motion therefore lever has constant angular velocity (ω)
- Measures force against a lever arm
- Moment = force times lever arm
- Instantaneous power = moment times angular velocity



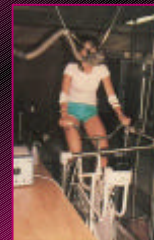
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Physiological Methods

- Oxygen Uptake
 - Difficult, accurate, expensive, invasive
- Physiological work = $c (VO_2)$
- Where, c is the energy released by metabolizing O_2 and VO_2 is the volume of O_2 consumed



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Mechanical Efficiency

- Measure both mechanical and physiological costs
- ME = mechanical cost divided by physiological cost times 100%

Monark ergometer used to measure mechanical work done



Mouthpiece for collecting expired gases and physiological costs

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Mechanical Efficiency

$$ME = \frac{\text{Internal work} + \text{External work}}{\text{Physiological cost}} \times 100\%$$

Internal work is measured by adding up the work done by all the joint moments of force. Most researchers ignore the internal work done.

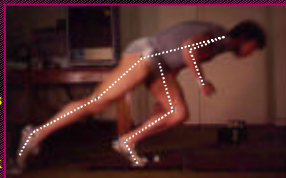
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Biomechanical Methods

- Total Mechanical Work = $\Sigma (\Sigma |M_j \omega_j| \Delta t)$
- Take absolute values of moment powers
- Sum over all joint moments and over duration of movement
- Internal work = Total work - external work



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