

# Fourier Analysis

## Fourier series:

$$w(t) = a_0 + \sum_{i=1}^n a_i \sin(\omega_i t + \theta_i)$$

where  $w(t)$  is the waveform,  $a_0$  is the dc-level,  $a_i$  are the amplitudes,  $\omega_i$  are the angular frequencies,  $t$  is the time and  $\theta_i$  are the phase angles. Note, the angular frequency  $\omega_i$  corresponds to frequency,  $f_i$ , such that,  $\omega_i = 2\pi f_i$ .

## Discrete Fourier Transform using Fourier Coefficients:

$$w(t) = a_0 + \sum_{i=1}^n [b_i \sin(\omega_i t) + c_i \cos(\omega_i t)]$$

## Equations of Known Waveforms

**Sine wave:**  $w(t) = a \sin \omega t$

### Square wave:

$$w(t) = a \left[ \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots \right]$$

### Triangle wave:

$$w(t) = \frac{8a}{\pi^2} \left[ \cos \omega t + \frac{1}{9} \cos 3\omega t + \frac{1}{25} \cos 5\omega t + \dots \right]$$

### Sawtooth wave:

$$w(t) = \frac{2a}{\pi} \left[ \sin \omega t - \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t - \frac{1}{4} \sin 4\omega t + \dots \right]$$