Analysis of Variance (ANOVA)

One-way ANOVA:

- used to test for significant differences among sample means
- differs from *t-test* since more than 2 groups are tested, simultaneously
- one factor (independent variable) is analyzed, also called the “grouping” variable
- dependent variable should be interval or ratio but independent variable is usually nominal

Factorial Design: - groups must be independent (i.e., subjects in each group are different and unrelated)

Assumptions:

- data must be normally distributed or nearly
- variances must be equal (i.e., homogeneity of variance)

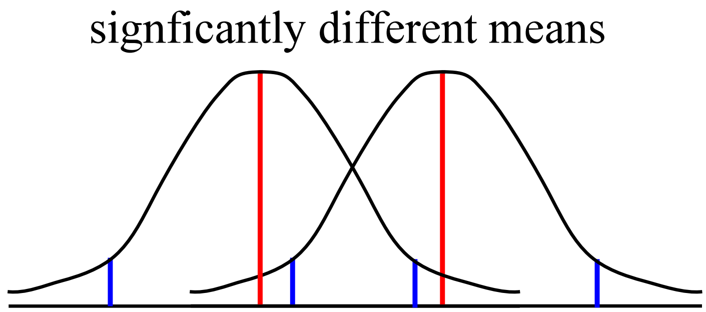
Examples:

- Does fitness level (VO_2max) depend on province of residence? Fitness level is a ratio variable, residence is a nominal variable.
- Does statistics grade depend of highest level of mathematics course taken?
- Does hand grip strength vary with gender? (Can be done with *t-test*. *t-test* can handle equal or unequal variances.)

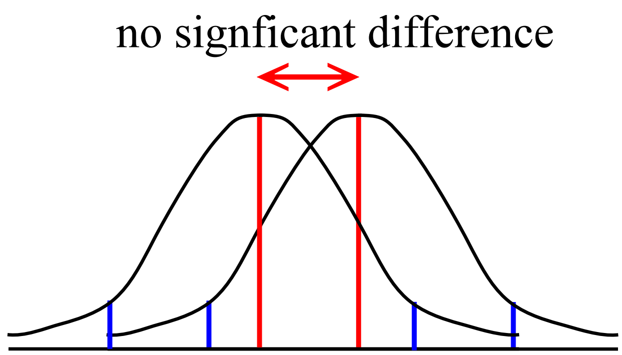
One-way ANOVA cont'd

An ANOVA tests whether one or more samples means are significantly different from each other. To determine which or how many sample means are different requires *post hoc* testing.

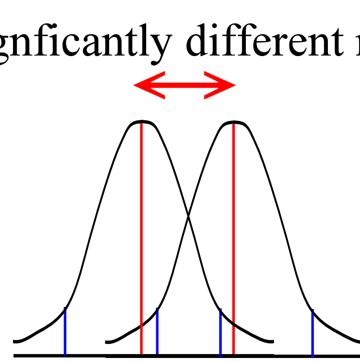
Two samples where means are significantly different.



These two sample means are NOT significantly different due to smaller difference and high variability.



Even with same difference between means, if variances are reduced the means can be significantly different.



One-way ANOVA cont'd

Step 1

H_0: all sample means are equal
H_1: at least one mean is different

Step 2

Find critical value from *F* table (Table A-5 or H). Tables are for one-tailed test. ANOVA is always one-tailed.

Step 3

Compute test value from:

$$F = \frac{s_B^2}{s_W^2} = \frac{\dfrac{\sum n_i(\overline{X}_i - \overline{X}_{GM})^2}{k-1}}{\dfrac{\sum (n_i - 1)s_i^2}{\sum (n_i - 1)}}$$

Step 4

Make decision.
If $F >$ critical value reject H_0.

Step 5

Summarize the results with ANOVA table.

All means are the same, i.e., come from the same population **or** at least one mean is significantly different.

Step 6 If a significant difference is found, perform *post hoc* testing to determine which mean(s) is/are different.

ANOVA Summary Table

Source	Sums of squares	*df*	Mean square	*F*	*P*
Between (also called Main effect)	SS_B	k–1	$SS_B/(k-1) = s_B^2$	s_B^2/s_W^2	
Within (also called Error term)	SS_W	N–k	$SS_W/(N-k) = s_W^2$		
Total	$SS_B + SS_W$	$(k-1)+(N-k)=N-1$			

Examples:

One-way Factorial

Source	Sums of squares	*df*	Mean square	*F*	*P*
Between	160.13	2	80.07	9.17	<0.01
Within	104.80	12	8.73		
Total	264.93	14			

Two-way Factorial

Source	Sums of squares	*df*	Mean square	*F*	*P*
Factor A	3.920	1	3.920	4.752	NS
Factor B	9.690	1	9.680	11.733	<0.025
A x B	54.080	1	65.552	79.456	<0.005
Within	3.300	4	0.825		
Total	70.980	7			

Post Hoc Testing

Post Hoc testing

- used to determine which mean or group of means is/are significantly different from the others
- many different choices depending upon research design and research question (Bonferroni, Duncan's, Scheffé's, Tukey's HSD, ...)
- **only done when ANOVA yields a significant *F***

Scheffé test:

- when sample sizes are unequal
- when most conservative test is desired

Critical value: Use critical value from ANOVA and multiply by k-1. k = number of groups (means)

$$F'_{critical} = (k-1)\, F_{critical}$$

Test value:

$$F_s = \frac{(\bar{X}_i - \bar{X}_j)^2}{s_W^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Decision:

If $F_s > F'_{critical}$, then the two means are significantly different.

Summary:

Graph the sample means and summarize.

Post Hoc Testing cont'd

Bonferroni test:

- used when less conservative test is desirable, i.e., more powerful
- may be used with other types of statistical tests (e.g., multiple *t*-tests)
- when only some pairs of sample means are to be tested

Critical value:

Use Table A-3 or F, adjust by dividing by number of all possible pairings.

Test value:

$$t = \frac{\overline{X}_i - \overline{X}_j}{\sqrt{s_w^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$$

Note, this is the same as taking the square root of the Scheffé test value.

Decision:

If $t > t_{critical}$, then the means are significantly different.

Summary:

Graph the results and summarize.

Post Hoc Testing cont'd

Tukey HSD test:

- sample sizes must be **equal** but a revised version permits unequal sample sizes (i.e., Tukey-Kramer)
- used when less conservative test is desirable, i.e., more powerful
- when all pairs of sample means are to be tested

Critical value:

Use Table N, where k = number of groups and v = degrees of freedom of $\mathbf{s_w}^2$

Test value: $$q = \frac{\overline{X}_i - \overline{X}_j}{\sqrt{\frac{s_W^2}{n}}}$$

Decision:

If q > critical value, then the means are significantly different.

Summary:

Graph the results and summarize.