Nonparametric Statistics

Nonparametric or Distribution-free statistics:

- used when data are ordinal (i.e., rankings)
- used when ratio/interval data are not normally distributed (data are converted to ranks)
- for studies not involving population parameters

Advantages:

- no assumptions about distribution of data
- suitable for ordinal data
- can test hypotheses that do not involve population parameters
- computations are easier to perform
- results are easier to understand

Disadvantages:

- less powerful (less sensitive) than parametric
- uses less information than parametric (ratio data are reduced to ordinal scale)
- less efficient, therefore larger sample sizes are needed to detect significance

Examples:

- Is there a bias in the rankings of judges from different countries?
- Is there a correlation between the rankings of two judges?
- Do different groups rank a professor's teaching differently?

Sign Test

- used to test claims about the median of a single population

Step 1

H_0: median is equal to a particular number
H_1: median is greater/less than that number

Step 2

Find critical value from table (Table A-7) for given level, sample size (>7) and use one-tailed hypothesis

Step 3

Subtract scores from the median then count number of positive (+) AND negative (–) differences. Zeros (pairs are equal) do not count. Test value is smaller count.

Step 4

Make decision. If smallest count (+ or –) is less than or equal to the critical value, reject H_0.

Step 5

Summarize the result.

I.e., there was/was not a change or there was/was not and increase/decrease in the dependent variable.

Paired Sign Test

- used for repeated measures tests

Step 1

H_0: no change/increase/decrease between before and after tests
H_1: there was a change/increase/decrease

Step 2

Find critical value from table (Table A-7) for given level, sample size (>7) and whether one- or two-tailed hypothesis

Step 3

Subtract "after" from "before" scores, then count number of positive (+) AND negative (–) differences. Zeros (pairs are equal) do not count. Test value is smaller count.

Step 4

Make decision. If smallest count (+ or –) is less than or equal to the critical value, reject H_0.

Step 5

Summarize the result.

I.e., there was/was not a change or there was/was not and increase/decrease in the dependent variable.

Wilcoxon Rank-Sum Test

- also called **Mann-Whitney U test**
- used to compare two independent groups
- replacement for independent groups t-test

Step 1 H_0: no difference/increase/decrease in group medians
H_1: there was a difference/increase/decrease between group medians

Step 2 Find critical value from *z*-table (Table A-2) for given -level and whether hypotenuse was one- or two-tailed.

Step 3

- Rank all data together.
- Sum ranks of group with <u>smaller</u> size (n_1). Call this R.
- Compute test value, z. (Note, n_1 and n_2 must be 10 or greater and n_2 is larger of two sample sizes or equal to n_1)

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$s_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

$$z = \frac{R - \mu_R}{s_R}$$

Step 4 If test value (z) > critical value, reject H_0.

Step 5 Summarize the result.

Wilcoxon Signed-Rank Test

- more powerful than Paired Sign Test
- use to compare two dependent samples (e.g., repeated measures)
- replaces dependent groups t-test

Step 1 H_0: no change/increase/decrease between groups
H_1: there is a change/increase/decrease

Step 2 Find critical value from (Table A-8) for given level, sample size (5 or greater) and whether hypothesis was one- or two-tailed. Use z-table (Table A-2) and *z* test value if n>30.

Step 3

- compute differences
- find absolute value of differences.
- rank the differences
- sum the positive and negative ranks separately and call smaller absolute value the test value, ***T***
- If *n* > 30 use test value, *z*, from:

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

Step 4 If ***T*** less than or equal to critical value or if *z* > critical z value, reject H_0.

Step 5 Summarize the result.

Kruskall-Wallis Test

- similar to Wilcoxon Rank-Sum test but for more than 2 groups
- replacement for One-way ANOVA

Step 1 H_0: there is no difference in medians
H_1: at least one group's median is different

Step 2 Find critical value from ^{2}-table (Table A-4) for given level and degrees of freedom (k-1). Test is always one-tailed (right-tailed).

Step 3

- Rank all data together.
- Sum ranks within each group, call them $R_1, R_2, R_3, \ldots, R_k$.
- Compute test value, H, from:

$$H = \frac{12}{N(N+1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} \ldots \frac{R_k^2}{n_k}\right) - 3(N+1)$$

where, $N = n_1 + n_2 + n_3 + \ldots n_k$

Step 4

If test value (H) > critical value, reject H_0.

Step 5

Summarize the result. I.e., there is a difference in at least one sample.

Spearman Correlation

- similar to Pearson except data are ordinal vs. ratio/interval
- data are ranked then correlated

Step 1 H_0: = 0
H_1: ≠ 0

Step 2 Find critical value from table (Table A-9) for given -level and sample size, *n* (number of pairs), *n* must be greater than 5 for =0.05

Step 3

- Rank data within each group
- Compute differences between pairs, $\mathbf{D}_i$
- Compute correlation coefficient from:

$$r_s = 1 - \frac{6\Sigma D_i^2}{n(n^2 - 1)}$$

where ***n*** is number of pairs (5 or more)

Step 4

If absolute value of test value ($|r_s|$) > critical value, reject H_0.

Step 5 Summarize the result. I.e., data are correlated or uncorrelated. Note, no regression line is possible since data were converted to ranks.