INTRODUCTION to BIOMECHANICS for HUMAN MOTION ANALYSIS, SECOND EDITION

SOLUTIONS to ODD-NUMBERED PROBLEMS

by

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INTRODUCTION (p. 12)

Conversion factors are taken from Table 1.3 on page 8.

1.

(a) $350 \times 0.4536 \times 9.81 = 1557$ N

(b) $6.50 / 0.4536 = 14.33$ lbs.

(c) $168.5 \times 2.54 = 428$ cm

(d) $(10 \times 100) / 2.54 = 394$ inches

(e) $$70.0 \frac{\text{miles}}{\text{hour}} \times \frac{1.609\text{ km}}{1\text{ mile}} \times \frac{1\text{ hour}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 31.3\text{ m/s}$$

(f) $$80.0 \frac{\text{miles}}{\text{hour}} \times \frac{1.609\text{ km}}{1\text{ mile}} = 128.7\text{ m/s}$$

(g) $8.35 \times 12 \times 2.54 = 255$ cm

(h) $440 \times 0.9144 = 402$ m

(i) $(800 / 0.9144) \times 3 = 2620$ feet

(j) $50.0 \times 1.609 = 80.5$ km

(k) $25.0 / 1.609 = 15.54$ miles

(l) $3.00 \times 9.81 = 29.4$ newtons

3.

$$250 \frac{\text{lbs}}{1} \times \frac{0.4536\text{ kg}}{1\text{ lbs}} = 113.4\text{ kg}$$

$$W = mg = 113.4 \times 9.81 = 1112.5\text{ N}$$

Thus, the 1200 N person weighs more than the 250 lbs. person.

5.

$$45\text{ ft.} = 45 \times 12 = 540\text{ in.} = 540 \times 2.54 = 1371.6\text{ cm} = 13.716\text{ m}$$

Thus, 13.75 m is longer than 45 feet.

FUNDAMENTAL CONCEPTS (p. 26)

$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2}$$

$$y = r\sin\theta \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

1.

(a) $r = \sqrt{250^2 + 350^2} = 430\text{ cm}$
$\theta = \tan^{-1}(350/250) = 54.5^\circ$

(b) $r = \sqrt{2.00^2 + 1.000^2} = 2.24\text{ kN}$
$\theta = \tan^{-1}(1.000/2.00) = 26.6^\circ$

(c) $r = \sqrt{-10.00^2 + -20.0^2} = 22.4\text{ m/s}$
$\theta = \tan^{-1}(-20.0/-10.00) = -116.6^\circ$

(d) $r = \sqrt{1000^2 + 2000^2} = 2240\text{ N}$
$\theta = \tan^{-1}(2000/1000) = 63.4^\circ$

(e) $r = \sqrt{25.0^2 + (-35.0)^2} = 43.0\text{ m/s}^2$
$\theta = \tan^{-1}(350/250) = -54.5^\circ$

(f) $r = \sqrt{-5.00^2 + 15.00^2} = 15.81\text{ kN}$
$\theta = \tan^{-1}(15.00/-5.00) = 108.4^\circ$

3.

(a) $x = 25.0\cos 25.0^\circ = 22.7\text{ m/s}$
$y = 25.0\sin 25.0^\circ = 10.57\text{ m/s}$

(b) $x = 10.00\cos 30.0^\circ = 8.66\text{ N}$
$y = 10.00\sin 30.0^\circ = 5.00\text{ N}$

(c) $x = 100.0\cos -45.0^\circ = 70.7\text{ m/s}^2$
$y = 100.0\sin -45.0^\circ = -70.7\text{ m/s}^2$

(d) $x = 5.00\cos(\pi/2) = 0.0\text{ m}$
$y = 5.00\sin(\pi/2) = 5.00\text{ m}$

(e) $x = 25.0\cos 160.0^\circ = -23.5\text{ N}$
$y = 25.0\sin 160.0^\circ = 8.55\text{ N}$

(f) $x = 25.0\cos(\pi/4) = 17.68\text{ m/s}$
$y = 25.0\sin(\pi/4) = 17.68\text{ m/s}$

5.

Sum of sides is always greater than length of hypotenuse.

7.

(a) $(30.0 - 20.0, 50.0 - 40.0) = (10.00, 10.00)\text{ cm}$

(b) $(400 - 500, 700 - 400) = (-100.0, 300)\text{ N}$

9.

R = (750, 1000) N = 1250 N at 53.1 deg

11.

$r = \sqrt{2.75^2 + 5.25^2} = 5.93\text{ km}$

RESOLUTION of FORCES into COMPONENTS (p. 33)

$$W = \text{weight} = mg \qquad F_G = \frac{Gm_1m_2}{r^2}$$
$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

1.

$$W = mg = 50.5 \times 9.81 = 495 \text{ N}$$

3.

$$150 \text{ lbs.} \times \frac{1 \text{ kg}}{2.2045 \text{ lbs}} = 68.2 \text{ kg}$$
$$\text{W} = \text{mg} = 68.2 \times 9.81 = 669 \text{ N}$$

5.

$$m = \frac{W}{g} = \frac{625}{9.81} = 63.7 \text{ kg}$$

7.

$$(a) \quad \underline{R} = (25 + 10 + 4.53, 30 + 12 - 1.56) = (39.5, 40.4)$$
$$(b) \quad \underline{R} = (3 \times 4.53 - 1.25, 3 \times -1.56 - 5) = (12.34, -9.68)$$
$$(c) \quad \underline{R} = (0 - 10 - 1.25, 10 - 12 - 5) = (-11.25, -7.00)$$
$$(d) \quad \underline{R} = (4.53 + 2(-1.25) - 0, -1.56 + 2(-5) - 10) = (2.03, -21.6)$$

$$(e) \quad \underline{R} = \left[\frac{1}{2}(4.53) + 5.6, \frac{1}{2}(-1.56 - 2)\right] = (7.87, -2.78)$$
$$(f) \quad \underline{R} = \left[\frac{1}{4}(2.53 + 10), \frac{1}{4}(30 + 12)\right] = (8.75, 10.50)$$
$$(g) \quad \underline{R} = (-1.25 - 10 + 0, -5 - 12 + 10) = (-11.25, -7.00)$$
$$(h) \quad \underline{R} = -[(10 - 25), (12 - 30)] = (15.00, 18.00)$$

9.

$$F_{moon} = \frac{1}{6}(9.81)60.0 = 98.1 \text{ N}$$
$$m_{space} = 60.0 \text{ kg}$$
$$F_{space} = 0.00 \text{ N}$$

11.

$$g = \frac{G \times m_{mars}}{r_{mars}^2}$$
$$= 6.673 \times 10^{-11} \times \frac{6.419 \times 10^{23}}{(3.396 \times 10^6)^2}$$
$$= 3.71 \text{ m/s}^2$$

MOMENT of FORCE (p. 42-3)

$$M = Fd$$
$$M = rF \sin\theta$$

1.
$$F = \frac{M}{d} = \frac{500}{1.650} = 303 \text{ N}$$

3. (a)
$$R = \Sigma F = -50.0 - 25.0 + 125.0 - 50.0 = 0.00 \text{ N}$$

(b)
$$M_R = \Sigma F_i d_i = 50.0(0.0) - 25.0(0.05) + 125.0(0.15) - 50.0(0.25)$$
$$= 0 - 1.25 + 18.75 - 12.50 = 5.00 \text{ N.m}$$

(c)
$$\Sigma M_A = 0$$
$$\therefore 125.0(d_C) - 50.0(0.0) - 25.0(0.05) - 50.0(0.25) = 0$$
$$125.0(d_C) = 0.0 + 25.0(0.05) + 50.0(0.25)$$
$$d_C = (1.25 + 12.50)/125.0 = 0.1100$$

The force at C should be moved 11.00 cm to the right of A.

5. (a)
$$\Sigma M = 200(2.50) - 250(2.20) = -50.0$$

Therefore, Cathy will go up.

(b) Jill must move 20.0 cm towards Cathy so the two moments are equal.

(c)
$$\Sigma M = 0$$
$$\therefore 200(2.50) - 250(2.20) - 150.0(d) = 0$$
$$d = \frac{-50.0}{-150.0} = 0.333$$

Therefore, Ian must sit 33.3 cm from fulcrum on Cathy's side.

7. (a) $M_A = Fd = -300(0.25) = -75.0 \text{ N.m}$

(b) $M_A = F_C \cos\theta(0.20) = -200(\cos 20^{O})(0.20) = -35.6 \text{ N.m}$

Note, vertical component at C cancels vertical component at D and horizontal component at D has no moment about axis at A.

9. $M = Fd = 56.5(0.650) = 36.7 \text{ N.m}$

LAWS of STATICS (p. 58-60)

$$\Sigma F_x = 0 \qquad \underline{M} = \underline{r} \times \underline{F} = (r_x F_y - r_y F_x)\underline{k}$$
$$\Sigma F_y = 0 \qquad \Sigma \underline{F} = \underline{0}$$
$$\Sigma M_A = 0 \qquad \Sigma \underline{M} = \Sigma(\underline{r} \times \underline{F}) = \underline{0}$$

1. (a)

$$\underline{M} = (0.35 \times 80.0 - 0.20 \times 50.0) = 18.00\underline{k} \text{ N.m}$$

(b)

$$\underline{M} = (0.35 \times 20.0 - 0.20 \times -30.0) = 13.00\underline{k} \text{ N.m}$$

(c)

$$\underline{M} = [-0.10 \times (80.0 + 20.0) - 0.30 \times (50.0 - 30.0)] = -16.00\underline{k} \text{ N.m}$$

(d)

$$M = (-0.10 \times 20.0 - 0.30 \times 20.0) = 7.00 \text{ N.m}$$

(e)

$$M = (1.250 \times -50.0 - 2.50 \times 85.0) = -275 \text{ N.m}$$

(f)

$$\underline{M} = (-0.10 \times -50.0 - 0.30 \times 85.0) = -20.5\underline{k} \text{ N.m}$$

3.

$$\underline{F} = (F\cos\theta, F\sin\theta) = (56.3\cos 34.3^\circ,\ 56.3\sin 34.3^\circ) = (46.51, 31.73)$$
$$\underline{M} = (0.1366 \times 31.73 - 0.205 \times 46.51) = -5.20\underline{k} \text{ N.m}$$

5. (a)

$$M = (4.00 \times 35.0 - 8.00 \times 25.0) = -60.0 \text{ N.cm} = -0.600 \text{ N.m}$$

(b)

$$M = (-4.00 \times 35.0 - 8.00 \times 25.0) = -340 \text{ N.cm} = -3.40 \text{ N.m}$$

(c)

$$M = (4.00 \times 35.0 - (-8.00) \times 25.0) = 340 \text{ N.cm} = -3.40 \text{ N.m}$$

(d)

$$M = (-4.00 \times 35.0 - (-8.00) \times 25.0) = 60.0 \text{ N.cm} = 0.600 \text{ N.m}$$

7.

$$C = \frac{L \times F_A}{mg} = \frac{2.00 \times 300}{60.0(9.81)} = 1.019 \text{ m} = 101.9 \text{ cm}$$

9.

$$\Sigma F_x = F_{g_x} + F_{knee_x} = 0$$
$$\Sigma F_y = F_{g_y} + F_{knee_y} - mg = 0$$
$$\Sigma M_{cg} = M_{knee} + (\underline{r}_{knee} \times \underline{F}_{knee}) + (\underline{r}_g \times \underline{F}_g) = 0$$

15.

$$\Sigma F_y = 0: \quad 2F_{cable} - mg = 0$$

$$F_{cable} = \frac{80.0(9.81)}{2} = 392 \text{ N per cable}$$

17. (a)

jacking up a car, prying with a bottle opener, shoveling

(b)

throwing a dart, kicking, jumping

21.

$$\Sigma F_x = 0: \quad -F_{1_x} + F_{2_x} = 0$$

$$F_{1_x} = F_{2_x}$$

$$F_{2_x} = 200\cos 30^\circ = 173.2 \text{ N}$$

$$\Sigma F_y = 0: \quad F_{1_y} + F_{2_y} - W = 0$$

$$F_{1_x} = F_{2_x} + mg = -200\sin 30^\circ + 400 = 300 \text{ N}$$

23.

$$\Sigma F_x = 0: \quad F_{knee_x} - F_{load_x} = 0$$

$$F_{knee_x} = F_{load_x}$$

$$F_{knee_x} = 250\cos 30^\circ = 217 \text{ N}$$

$$\Sigma F_y = 0: \quad F_{knee_y} - F_{load_y} - W = 0$$

$$F_{knee_x} = F_{load_x} + mg = 250\sin 30^\circ + 40.0 = 165 \text{ N}$$

DRY FRICTION (pp. 71-2)

$$F_{static} = \mu_{static} F_{normal}$$
$$F_{kinetic} = \mu_{kinetic} F_{normal}$$

1.

$$\Sigma F_n = 0: \quad F_{normal} - mg = 0$$
$$F_{normal} = mg = 50 \times 9.81 = 490.5$$
$$F_{static} = \mu_{static} F_{normal} = 0.95 \times 490.5 = 466 \text{ N}$$
$$F_{kinetic} = \mu_{kinetic} F_{normal} = 0.90 \times 490.5 = 441 \text{ N}$$

3.

$$\Sigma F_n = 0: \quad F_{normal} - mg = 0$$
$$F_{normal} = mg = 35 \times 9.81 = 343.4$$

Since object is in motion :

$$F_{kinetic} = \mu_{kinetic} F_{normal} = 0.90 \times 343.4 = 188.8 \text{ N in negative direction}$$

5.

Since there are only three forces and they must add to zero for statics you can apply the triangle rule :

$$\Sigma F = 0: \quad \underline{F}_{friction} + \underline{F}_{normal} + \underline{W} = 0$$

be constructing the triangle we see that :

$$F_{friction} = W \sin 15 = 50 \times 9.81 \times 0.25882 = 127.0 \text{ N up the incline}$$

7.

$$\Sigma F_n = 0:$$
$$F_{normal} - mg \cos 12^\circ + F_{applied} \sin 45^\circ = 0$$
$$F_{normal} = 30 \times 9.81 \times \cos 12^\circ - 200 \sin 45^\circ = 146.4$$
$$F_{static} = \mu_{static} F_{normal} = 0.80 \times 146.4 = 117.2$$

Assume $\Sigma F_t = 0$:

$$F_{equilbrium} - mg \sin 12^\circ + F_{applied} \cos 45^\circ = 0$$
$$F_{equilbrium} = 30 \times 9.81 \times \sin 12^\circ - 200 \cos 45^\circ = -80.2$$

since this absulute value is smaller than F_{static} friction is 80.2 N down the incline.

9.

$$\Sigma F_n = 0: \quad F_{normal} - mg - F_{applied} \sin 13^\circ = 0$$
$$F_{normal} = 50 \times 9.81 + 500 \sin 13^\circ = 603.0$$
$$F_{static} = \mu_{static} F_{normal} = 0.80 \times 603 = 482.4 \text{ N}$$

Assume $\Sigma F_t = 0: \quad F_{equilbrium} + F_{applied} \cos 13^\circ = 0$

$$F_{equilbrium} = -500 \cos 13^\circ = -487.2$$

since absolute value is greater than F_{static} the body is moving (friction = $F_{kinetic}$).

11.

$$\Sigma F_n = 0: \quad F_{normal} - W\cos 10^\circ = 0$$

$$F_{normal} = 250\cos 10^\circ = 246.2$$

$$F_{static} = \mu_{static} F_{normal} = 0.500 \times 246.2 = 123.10 \text{ N}$$

To get the box moving the friction must equal F_{static}.

$$\Sigma F_t = 0: \; -F_{static} - W\cos 10^\circ + F_{applied} = 0$$

$$F_{applied} = F_{static} + 250\sin 10^\circ = 123.10 + 43.41 = 166.5 \text{ N}$$

13.

$$F_{static} = \mu_{static} F_{normal} = 0.950(150.0 \times 9.81) = 1398 \text{ N}$$

$$F_{kinetic} = \mu_{kinetic} F_{normal} = 0.900(150.0 \times 9.81) = 1324 \text{ N}$$

15.

$$F_{normal} = 10.00 \times 9.81 = 98.1 \text{ N}$$

$$\mu_{static} = \frac{F_{static}}{F_{normal}} = \frac{25.0}{98.1} = 0.255$$

$$\mu_{kinetic} = \frac{F_{kinetic}}{F_{normal}} = \frac{20.0}{98.1} = 0.204$$

17.

$$F_{normal} = 30.0 \times 9.81 = 294.3 \text{ N}$$

$$\mu_{static} = \frac{F_{static}}{F_{normal}} = \frac{225}{294.3} = 0.765$$

$$\mu_{kinetic} = \frac{F_{kinetic}}{F_{normal}} = \frac{215}{294.3} = 0.731$$

LINEAR KINEMATICS (pp. 87-8)

$$v_f = v_i + at$$
$$s_f = s_i + v_i t + \tfrac{1}{2}at^2$$
$$v_f^2 = v_i^2 + 2a(s_f - s_i)$$
$$s_f = s_i + \tfrac{1}{2}(v_i + v_f)t$$

1. (a)

0 to 40 m: $a = \dfrac{v_f^2 - v_i^2}{2(s_f - s_i)}$

$$a = \frac{9^2 - 0}{2(40-0)} = \frac{9^2}{2(40)}$$
$$= \frac{81}{80} = 1.0125 \text{ m/s}^2$$

40 to 70 m: $a = 0 \text{ m/s}^2$

70 to 100 m: $s_f = s_i + vt + \tfrac{1}{2}at^2$

$$100 = 70 + 9(4) + \frac{1}{2}a(4)^2$$
$$a = 2\frac{(30-36)}{16} = -0.750 \text{ m/s}^2$$

(b)

$$t_{0-40} = \frac{v_f - v_i}{a} = \frac{9-0}{1.0125} = 8.89 \text{ s}$$
$$t_{40-70} = \frac{70-40}{9} = 3.33 \text{ s}$$
$$t_{70-100} = 4.00 \text{ s}$$
$$t_{total} = 8.89 + 3.33 + 4 = 16.22 \text{ s}$$

(c)

$$s_f = 0 + 0 + \frac{1}{2}\left(\frac{81}{80}\right)5^2 = 12.66 \text{ m}$$

(d)

$$v_{100} = 9 - \left(\frac{3}{4}\right)4 = 6.00 \text{ m/s}$$

3. (a)

$$v_f{}^2 = \pm\sqrt{3.5^2 + 2(0.005)(25-0)}$$
$$= \pm 3.536 \text{ m/s}$$
$$t = \frac{v_f - v_i}{a} = \frac{+3.536 - 3.50}{0.005} = 7.11 \text{ s}$$

(b)

$$s_f = s_i + v_i t + \frac{1}{2}at^2$$
$$= 0 + 3.5\times 10^2 + \frac{1}{2}(0.005)10^2$$
$$= 35 + 0.25 = 35.3 \text{ m}$$

5. (a)

$$t = \frac{v_f - v_i}{a} = \frac{0 - 2.20}{-0.300} = 7.33 \text{ s}$$

(b)

$$s_f = s_i + v_i(7.333) + \frac{1}{2}at^2$$
$$= 0 + 2.20(7.333) +$$
$$\frac{1}{2}(-0.3)(7.33)^2 = 8.07 \text{ m}$$

7.

$$v = 150 \text{ km/h} \div 3.6 = 41.67 \text{ m/s}$$

1^{st} catcher:

$$a = \frac{v_f^2 - v_i^2}{2(s_f - s_i)} = \frac{0 - 41.67^2}{2(0.300-0)}$$
$$= -2894$$
$$t = \frac{v_f - v_i}{a} = \frac{0 - 41.67}{-2894}$$
$$= 0.01440 \text{ s} = 14.40 \text{ ms}$$

2^{nd} catcher:

$$a = \frac{v_f^2 - v_i^2}{2(s_f - s_i)} = \frac{0 - 41.67^2}{2(0.500-0)}$$
$$= -1736$$
$$t = \frac{v_f - v_i}{a} = \frac{0 - 41.67}{-1736}$$
$$= 0.0240 \text{ s} = 24.0 \text{ ms}$$

The difference in the accelerations is 1158 m/s^2. The difference in the times is 9.60 ms.

9.

$$s = \sqrt{12.00^2 + \left(\frac{7.32}{2}\right)^2} = 12.546 \text{ m}$$

$$v = s / t = 12.546 / 1.500 = 8.36 \text{ m/s}$$

Ball speed must be greater than 8.36 m/s.

11. (a)

$$v = \frac{s}{t} = \frac{100 \text{ m}}{4 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 0.417 \text{ m/s}$$

(b)

$$s_{down\ river} = v_{current} t = 4.00 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times 4.00 \text{ min} = 266.7 \text{ m}$$

$$s_{total} = \sqrt{266.7^2 + 100.0^2} = 285 \text{ m}$$

(c)

$$v = s_{total} / t = 284.8 / (4.00 \times 60.0) = 1.187 \text{ m/s}$$

13.

$$0 = v_i^2 + 2a(s_f - s_i)$$

$$v_i^2 = -2a(s_f - s_i) = -2(-2.00)(6.00 - 0)$$

$$v_i = \pm\sqrt{24.0} = \pm 4.90$$

The initial velocity must be 4.90 m/s.

PROJECTILE MOTION (pp. 94-5)

$$v_{fy} = v_{iy} - gt$$
$$s_{fy} = s_{iy} - gt$$
$$v_{fy}^2 = v_{iy}^2 - 2g(s_{fy} - s_{iy})$$
$$s_{fy} = s_{iy} + \frac{1}{2}(v_{iy} + v_{fy})t$$

1. (a)

$$90\frac{km}{h} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 25.0\text{ m/s}$$
$$v_x = v\cos\theta = 25\cos 5° = 24.90\text{ m/s}$$
$$v_y = v\sin\theta = 25\sin 5° = 2.179\text{ m/s}$$

(b)

$$s_{fx} = s_{ix} - v_x t$$
$$t = \frac{10-0}{24.9} = 0.402s$$

(c)

$$v_{fy}^2 = v_{iy}^2 - 2g(s_{fy} - s_{iy})$$
$$s_{fy} = \frac{v_{fy}^2 - v_{iy}^2}{-2g} + s_{iy}$$
$$= \frac{0-(2.179)^2}{-2(9.81)} + 0 = 0.242\text{ m} = 24.2\text{ cm}$$

(d)

$$v_x = \frac{10-0}{0.350} = 28.6\text{ m/s}$$

3. (a)

$$v_{fy}^2 = v_{iy}^2 - 2g(s_{fy} - s_{iy})$$
$$v_{fy} = \pm\sqrt{0-2(9.81)(-20-0)}$$
$$= \pm\sqrt{392.4} = \pm 19.81\text{ m/s}$$
$$v_f = -19.81\text{ m/s}^2$$

(b)

$$t = \frac{v_f - v_i}{-g} = \frac{-19.81-0}{-9.81} = 2.02\text{ s}$$

(c)

$$s_{tx} = s_{ix} - v_x t = 0 + 2(2.02) = 4.04\text{ m}$$

5. (a)

$$s_{fy} = s_{iy} + \frac{v_{fy}^2 - v_{iy}^2}{-2g}$$
$$= 0 + \frac{0 + v_{iy}^2}{+2g} = \frac{6.00^2}{2(9.81)} = 1.835\text{ m}$$

(b)

$$t = \frac{v_f - v_i}{-g}$$
$$v_f^2 = v_i^2 - 2g(s_f - s_i)$$
$$= 6^2 - 2(9.81)(0-10)$$
$$v_f = \pm\sqrt{232.2} = \pm 15.24\text{ m/s}$$

select the negative velocity

$$t = \frac{-15.24-6}{-9.81} = 2.16\text{ s}$$

(c)

$$s_x = v_x t = 0.500(2.16) = 1.082\text{ m}$$

7.

θ	v_x	v_y	y_{max}	x_{max}	time
30°	8.66	5.00	1.270	8.83	1.019
45°	7.07	7.07	2.55	10.19	1.442
60°	5.00	8.66	3.82	8.83	1.765

9.

$$v_{fy}^2 = 0 - 2g(1.0-1.5) = 9.81$$
$$v_{fy} = \pm\sqrt{9.81} = -3.13$$
$$t = \frac{v_f - v_i}{-g} = \frac{-3.13-0}{-9.81} = 0.319\text{ s}$$
$$v_x = \frac{10}{0.319} = 31.3\text{ m/s}$$

11.

$$v_f^2 = 0 - 2(9.81)(-0.795)$$
$$v_f = -3.94\text{ m/s}$$
$$t = \frac{v_f}{g} = 0.4026$$
$$v_{fps} = \frac{12}{0.4206} = 29.8\text{ fps}$$

ANGULAR KINEMATICS (p. 99)

$$\omega_f = \omega_i + \alpha t$$
$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$
$$\theta_f = \theta_i + \tfrac{1}{2}(\omega_i + \omega_f)t$$

1. (a)

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{5-2}{2} = 1.500 \text{ r/s}^2$$
$$= 3\pi \text{ rad/s}^2 = 9.42 \text{ rad/s}^2$$

(b)

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$= 0 + 2(2) + \frac{1}{2}(1.5)2^2 = 7.00 \text{ revolutions}$$

3.

$$\theta = 0 + 0\frac{1}{2}(0.35)36^2$$
$$= 227 \text{ revolutions}$$

5.

$$\omega_f = \omega_i + \alpha t$$
$$= 6 + 0.750(5) = 9.75 \text{ rad/s}$$

7.

$$\omega = \frac{3r}{0.89} = \frac{6\pi}{0.89} = 21.2 \text{ rad/s}$$

9.

$$\omega_{final} = \omega_{initial} + \alpha t = 5.60 + (-0.200)3.00 = 5.00 \text{ rad/s}$$

11.

$$t = \frac{\omega_{final} - \omega_{initial}}{\alpha} = \frac{0 - 20.0}{-1.500} = 13.33 \text{ s}$$

RELATIONSHIP between LINEAR and ANGULAR MEASURES (pp. 104-5)

$$v_t = r\omega$$
$$a_t = r\alpha$$
$$a_r = r\omega^2 = \frac{v_t^2}{r}$$
$$a = \sqrt{a_r^2 + a_t^2}$$

1. (a)

$$v_t = r\omega = 0.75(15) = 11.25 \text{ m/s}$$

(b)

$$a_t = r\alpha = 0.75(150) = 117.5$$
$$a_r = r\omega^2 = 0.75(15)^2 = 168.75$$
$$a = \sqrt{{a_t}^2 + {a_r}^2} = \sqrt{117.5^2 + 168.75^2}$$
$$= 203 \text{ m/s}^2$$

3. (a)

$$v_t = r\omega = 0.900\left(573\frac{\text{deg}}{s}\right)\times\frac{\pi \text{ rad}}{180 \text{ deg}}$$
$$= 9.00 \text{ m/s}$$

(b)

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{10-0}{1.5} = 6.67 \text{ rad/s}^2$$

(c)

$$a_t = r\alpha = 0.9(10) = 9.00$$
$$a_r = r\omega^2 = 0.9(10)^2 = 90.0$$
$$a = \sqrt{{a_t}^2 + {a_r}^2} = \sqrt{9.00^2 + 90.0^2}$$
$$= 90.4 \text{ m/s}^2$$

5.

$$v_t = r\omega$$
$$= 0.75(10.00)$$
$$= 7.50 \text{ m/s}$$
$$a_r = r\omega^2 = 0.75(10)^2 = 75.0 \text{ m/s}^2$$
$$a_t = r\alpha = 0.75(2.00) = 1.500 \text{ m/s}^2$$
$$a = \sqrt{{a_t}^2 + {a_r}^2} = \sqrt{75^2 + 1.5^2}$$
$$= 75.0 \text{ m/s}^2$$

7. (a)

$$v_{tfeet} = r\omega = 2.30(10) = 23.0 \text{ m/s}$$
$$v_{tcg} = r\omega = 1.30(10) = 13.00 \text{ m/s}$$

(b)

$$\omega = \frac{v}{r} = \frac{10-0}{2} = 5.00 \text{ rad/s}$$

(c)

$$s_{feet} = 2\pi r = 2\pi(2.30) = 14.45 \text{ m}$$
$$s_{cg} = 2\pi r = 2\pi(1.3) = 8.16 \text{ m}$$
$$= 14.45 - 8.16 = 6.29 \text{ m}$$

(d)

$$a_{r\,feet} = r\omega^2 = 2.30(10.00)^2$$
$$= 230 \text{ m/s}^2$$

9. (a)

$$r_{total} = 1.00 + .70 = 1.700 \text{ m}$$
$$v_t = r\omega = 1.70(8.75) = 14.875 \text{ m/s}$$
$$v = r\omega;\ r = \frac{v}{\omega},$$
$$1.70 + r = \frac{v_t + 1.5}{\omega}$$
$$r = \frac{16.375}{8.75} - 1.7 = 0.1714 \text{ m}$$

(b)

$$a_r = r\omega^2 = 0.1714(8.75)^2$$
$$= 13.12 \text{ m/s}^2$$

11.

$$\omega = 65\frac{\text{r}}{\text{min}}\times\frac{2\pi \text{ rad}}{1 \text{ r}}\times\frac{1 \text{ min}}{60 \text{ s}}$$
$$= 6.81 \text{ rad/s}$$
$$v_{rim} = r\omega = 0.330(6.81) = 2.25 \text{ m/s}$$
$$v_{pushwheel} = r\omega = 0.100(6.81)$$
$$= 0.681 \text{ m/s}$$

LAW of ACCELERATION (p. 110)

$$\Sigma \underline{F} = m\,\underline{a}$$
$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$

1.

$$\mathrm{a} = \left(\frac{\mathrm{v_f^2} - \mathrm{v_i^2}}{2(\mathrm{x_f} - \mathrm{x_i})}\right) = \left(\frac{0 - 9^2}{2(30 - 0)}\right) = \frac{-81}{60} = -1.350\ \mathrm{m/s^2}$$
$$\mathrm{F} = \mathrm{ma} = 60.0(-1.350) = -81.0\ \mathrm{N}$$

3.

$$\Sigma F_y = ma_y = F_{lifter} - mg$$
$$a_y = \left(\frac{v_{yf} - v_{yi}}{t}\right) = \frac{2 - 0}{1} = 2.00$$
$$F_{lifter} = ma_y + mg = 40(2) + 40(9.81) = 472\ \mathrm{N}$$

5.

$$F_{normal} = mg = 70(9.81) = 686.7$$
$$F_{kinetic} = F_{normal}\mu_{kinetic} = 686.7 \times 0.6 = 412\ \mathrm{N}$$
$$\Sigma F_x = ma_x = -F_{kinetic}$$
$$a_x = \frac{-Fkinetic}{70} = -5.886\ \mathrm{m/s^2}$$
$$\text{Thus, } x_f = x_i + \left(\frac{v_{fx}^2 - v_{ix}^2}{2a_x}\right) = 0 + \left(\frac{0 - 10^2}{2(-5.886)}\right) = 8.49\ \mathrm{m}$$

7.

$$\Sigma F = ma = 1000 = 900a$$
$$a = 1000/900 = 1.111\ \mathrm{m/s^2}$$
$$t = \frac{v_f - v_i}{a} = \frac{3 - 0}{1.111} = 2.70\ \mathrm{s}$$

9.

$$a = \frac{v_f - v_i}{t} = \frac{0 - 20}{0.3} = -66.67\ \mathrm{m/s^2}$$
$$F = ma = 0.180(-66.67) = -12.00\ \mathrm{N}$$

11.

$$\Sigma F_x = ma_x = 35 + 58 - 52 = 41.0\ \mathrm{N}$$
$$\Sigma F_y = ma_y = 25 - 20 + 23 - mg = 28 - 196.2 = -168.2\ \mathrm{N}$$
$$a_x = 41/20 = 2.05\ \mathrm{m/s^2}$$
$$a_y = -168.2/20 = -8.41\ \mathrm{m/s^2}$$

MOMENT of FORCE (p. 115)

$$\Sigma M = I\,\alpha$$
$$M = F\,d$$

1.

$$\alpha = \frac{M}{I} = 35/2 = 17.50 \text{ rad/s}^2$$

3.

$$M = Fd = 150(0.35) = 52.5 \text{ N.m}$$
$$\alpha = M/I = 2.10 \text{ rad/s}^2$$

5.

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2.35 - 4.45}{5.00} = -0.420$$
$$M = I\alpha = 32.0(-0.42) = -13.44 \text{ N.m}$$

7.

$$M = Fd = 400 \times 0.35 = 140.0 \text{ N.m}$$

9.

$$M = Fd = 500 \times 0.40 = 200 \text{ N.m}$$
$$I = M/\alpha = 200/20 = 10.00 \text{ kg.m}^2$$

MOMENT of INERTIA (p. 122)

$$K_{cg} = k_{cg}/L$$
$$I_{cg} = mk_{cg}^2$$
$$I_{axis} = I_{cg} + mr^2$$

1.

$$k_{cg} = \sqrt{\frac{I_{cg}}{m}} = \sqrt{\frac{0.5}{8}} = 0.250 \text{ m}$$
$$I_{axis} = I_{cg} + mr^2 = 0.5 + 8(0.25)^2$$
$$= 1.000 \text{ kg.m}^2$$

3.

$$k = KL = 0.60(0.95) = 0.57 \text{ m}$$
$$I_{cg} = mk^2 = 12.00(0.57)^2 = 3.90 \text{ kg.m}^2$$
$$I_{hip} = I_{cg} + mr^2$$
$$= 3.90 + 12.00(0.50)^2 = 6.90 \text{ kg.m}^2$$

5. (a)

$$I_{bar} = I_{cg} + mr^2 = 15.80 + 80.0(1.450)^2$$
$$= 15.8 + 168.2 = 184.0 \text{ kg.m}^2$$

(b)

$$k = \sqrt{\frac{15.80}{80.0}} = \sqrt{0.1975} = 0.444 \text{ m}$$

7.

$$M = Fd = 300(0.15) = 45.0 \text{ N.m}$$
$$\alpha = (\omega_f - \omega_i)/t = 20/1 = 20.0 \text{ rad/s}^2$$
$$I = M/a = 45/20 = 2.25 \text{ kg/m}^2$$
$$k = \sqrt{I/m} = \sqrt{2.25/90.0} = 0.1581 \text{ m}$$

9.

$$I_{proximal} = I_{cg} + mr^2 = 0.1489 + 7.05(0.1925)^2$$
$$= 0.410 \text{ kg.m}^2$$
$$k_{proximal} = \sqrt{I_{proximal}/m} = \sqrt{0.453/7.05} = 0.241 \text{ m} = 2.41 \text{ cm}$$

LAW of REACTION (p. 129)

1.

$$\Sigma F_x = F_{gx} = ma_x$$

$$a_x = F_{gx} / m = 227 / 53.8 = 4.22 \text{ m/s}^2$$

$$\Sigma F_y = F_{gy} - mg = ma_y$$

$$a_y = (1345 - 53.8 \times 9.81) / 53.8 = 15.19 \text{ m/s}^2$$

3.

$$s = r\theta$$

$$r = s / \theta = 100 / \pi = 31.83$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \frac{11^2}{31.83 \times 9.81}$$

$$= \tan^{-1}(0.3875) = 21.2 \text{ deg}$$

5.

$$mg = m\frac{v^2}{r}$$

$$r = \frac{v^2}{g} = \frac{250^2}{9.81} = 6371 \text{ m} = 6.37 \text{ km}$$

7.

$$\Sigma Fy = ma_y = F_{gy} - mg$$

$$F_{gy} = ma_y + mg = 65.0(0.75 + 9.81)$$

$$= 686 \text{ N}$$

9.

$$ma_{\mathrm{r}} = -mv_{\mathrm{t}}^2 / r = -mr\omega^2$$

$$= -3.50(2.00)5^2 = -175.0 \text{ N}$$

11.

$$F = ma = 75(20 \times 9.81) = 14\,720 \text{ N}$$

LINEAR IMPULSE and MOMENTUM (p. 140)

$$\text{momentum} = p = mv$$
$$\text{impulse} = \int F dt = \overline{F}t$$
$$\int F dt = mv_f - mv_i$$

1. $\overline{F} = m\overline{a} = 60.0\left(\dfrac{0-12.00}{1.100}\right) = 60(-10.909) = -655 \text{ N}$

3. $\text{Impulse} = Ft = mv_f - mv_i = 55.0(2.50) - 0 = 137.5 \text{ N.s}$

5.

$$mv_{fx} = mv_{ix} + \int F_x dt = 0 + 200$$
$$v_{fx} = 200/70.0 = 2.86 \text{ m/s}$$
$$mv_{fy} = mv_{iy} + \int F_y dt - Wt$$
$$= 0 + 1200 - 70(9.81)1.2 = 375.86$$
$$v_{fy} = 375.86/70.0 = 5.37 \text{ m/s}$$

7.

$$v_{fy}^2 = v_{iy}^2 - 2g(y_f - y_i)$$
$$= 0 - 2(9.81)(-1.350 - 0) = 26.487$$
$$v_{fy} = \sqrt{26.476} = -5.1247 \text{ m/s}$$
$$a_{landing} = \frac{v_f - v_i}{t} = \frac{0-(-5.127)}{0.400} = 12.87 \text{ m/s}^2$$
$$F_{landing} = ma_{landing} = 60.0(12.87) = 772 \text{ N}$$

9.

$$\overline{F}t = mv_f - mv_i$$
$$\overline{F} = \frac{-mv_i}{t} = \frac{-0.005(400)}{0.050} = -40.0 \text{ N}$$

11.

$$v_f = v_i + \frac{\overline{F}t}{m} = 0 + 24.0(1)/4 = 6.00 \text{ m/s}$$

ANGULAR IMPULSE and MOMENTUM (p. 145)

$$\text{angular momentum} = L = I\omega$$
$$\text{angular impulse} = \int M dt = \overline{M}t$$
$$\int M dt = I\omega_f - I\omega_i$$

1. $L = I\omega = 5.00(5.52)$

$= 27.6\ \text{kg.m}^2/\text{s}$

3. $L = I\omega = 5.65\left(2.25\frac{r}{s} \times \frac{2\pi\ \text{rad}}{1\,r}\right)$

$= 5.65(14.137) = 79.9\ \text{kg.m}^2/\text{s}$

5. $\overline{M}t = (\overline{F}d)t = 250(1.350)0.500$

$= 168.8\ \text{N.m.s}$

7. $\overline{M}t = (\overline{F}d)t = 68.0(0.320)5$

$= 108.8\ \text{N.m.s}$

9. $k = Kl = 0.326(1.235) = 0.4026\ \text{m}$

$I = mk^2 = 11.50(0.4026)^2 = 1.8641\ \text{kg.m}^2$

$L = I\omega = 1.8641(2.55) = 4.75\ \text{kg.m}^2/\text{s}$

CONSERVATION of MOMENTUM (p. 153)

$$p_f = p_i = mv = \text{constant}$$
$$L_f = L_i = I\omega = \text{constant}$$

1. (a)

$$L = I\omega = 2(20) = 40.0\ \text{kg.m}^2/\text{s}$$

(b)

$$I_{top}\omega_{top} = L_{start} = L_{top} = L_{land} = 40.0\ \text{kg.m}^2/s$$
$$I_{top} = L/30.0 = 1.333\ \text{kg.m}^2$$
$$I_{land} = L/26.6 = 1.504\ \text{kg.m}^2$$

3. (a)

$$L = I\omega$$
$$8.25 = 0.430\omega$$
$$\omega = 8.25/0.43 = 19.19\ \text{rad/s}$$

(b)

$$M = I\alpha = 0.43\left(\frac{19.19 - 0}{0.5}\right) = 16.50\ \text{N.m}$$

5.

$$\omega = L/I = 8.95/16.26 = 0.5504$$
$$\omega = \frac{\theta_f - \theta_i}{t}$$
$$t = \frac{\theta_f - 0}{\omega} = \frac{2p}{0.5504} = 11.42\ \text{s}$$

7.

$$I = L/\omega = 35.6/3.25 = 10.95\ \text{kg.m}^2$$

WORK-ENERGY THEOREM (p. 162)

$$\text{work} = W = \Delta E = E_f - E_i$$
$$\text{energy} = E = mgy + ½mv^2 + ½I\omega^2$$

1.
$$E = mgy + ½mv^2 = 70(9.81)1.150 + ½(70)12.00^2$$
$$= 789.7 + 5040 = 5830 \text{ joules} = 5.83 \text{ kJ}$$

3.
$$E = ½mv^2 = ½(60.0)2.00^2 = 120.0 \text{ J}$$

5.
$$E = ½\text{Iw}^2 = ½(2.00)\left(300\frac{r}{\min}\times\frac{2\pi}{1r}\times\frac{1\min}{60s}\right)^2$$
$$= ½(2.00)31.416^2 = 987 \text{ J}$$

7.
$$y = v\sin 20^\circ t = 6.00\sin 20^\circ(60.0 \text{ s}) = 123.1 \text{ m}$$
$$W = Ef - Ei = mgy_f - mgy_i$$
$$= 50(9.81)123.1 - 0 = 490.5(123.1) = 60.4 \text{ kJ}$$

9.
$$W = E_f - E_i = 0 - E_i = 0 - ½mv^2 = -½(25.0)5^2 = -313 \text{ J}$$

11. (a)
$$a = \frac{vf - vi}{t} = \frac{0-3}{4} = -0.750$$
$$s_f = s_i + \frac{v_f^2 - v_i^2}{2a} = \frac{0-3^2}{2(-0.75)} = \frac{-9.0}{-1.50} = 6.00 \text{ m}$$

(b)
$$\overline{F} = m\overline{a} = 20.0(-0.75) = -15.00 \text{ N}$$

(c)
$$W = Fs = -15.00(6.00) = -90.0 \text{ J}$$

WORK of a FORCE or MOMENT of FORCE (p. 168-9)

$$W_{force} = Fs\cos\phi$$
$$W_{force} = \underline{F}\cdot\underline{s} = F_x s_x + F_y s_y$$
$$W_{moment} = M\theta = (Fr\sin\phi)\theta$$

1. (a)

$$W = Fs = 358\times 0.517 = 185.1\text{ J}$$

(b)

$$W = F_x s_x + F_y s_y$$
$$= 25.3\times 1.325 + 63.2\times 2.92$$
$$= 218\text{ J}$$

3.

$$W_{force} = Fs\cos\phi$$
$$= 35.0\,(23.0)\cos 30^\circ = 697\text{ J}$$

5.

$$F_{normal} = mg = 25.0(9.81)$$
$$= 245.25\text{ N}$$
$$F_{kinetic} = F_{normal}\mu_{kinetic}$$
$$= 245.25\times 0.800 = 196.2\text{ N}$$
$$W_{\mu=0.8} = F_{kinetic}s = 196.2\times 10.00$$
$$= 1962\text{ J}$$
$$W_{\mu=0.2} = 245.25\times 0.2\times 10.00$$
$$= 491\text{ J}$$

7.

$$W = Fs = (Lg)s$$
$$= 20(9.81)10(6.00)$$
$$= 196.2(60.0) = 11772\text{ J}$$
$$= 11.77\text{ kJ}$$

9.

$$W = Fs = (Lg)s$$
$$= 3.50(9.81)4000$$
$$= 137\,340\text{ J} = 137.3\text{ kJ}$$

11.

$$W = mgy = 60(9.81)0.350$$
$$= 206\text{ J}$$

13. (a)

$$W_{total} = mgy = 300(9.81)0.1500$$
$$= 441\text{ J}$$

(b)

$$W_{moment} = M\theta = (Fd)\theta$$
$$= (90.0\times 0.600)(1\text{ rad}) = 54.0\text{ J}$$

(c)

$$n = W_{total}/W_{moment}$$
$$= \frac{441}{54} = 8\text{ and }1/6 = 8.17\text{ cycles}$$

15. (a)

$$W = E_f - E_i = 0 - \tfrac{1}{2}I\omega^2$$
$$= -\tfrac{1}{2}(0.450)\,20.0^2 = -90.0\text{ J}$$

(b)

$$F_{friction} = F_{normal}\mu_{kinetic}$$
$$= -12.50(0.800) = -10.00\text{ N}$$
$$\alpha = \frac{M}{I} = \frac{Fd}{I} = \frac{10.00\times 0.300}{0.450}$$
$$= -6.667\text{ rad/s}^2$$
$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0-20.0}{-6.667} = 3.00\text{ s}$$

POWER (p. 173)

$$P = W/t = \Delta E/t$$
$$P_{force} = Fv\cos\phi$$
$$P_{force} = \underline{F} \cdot \underline{v} = F_x v_x + F_y v_y$$
$$P_{moment} = M\omega$$

1.

$s = 2 \times 70.0 \times 6.00 = 840$ metres

$W_L = 10.00 \times 9.81 = 98.1$ joules

$P = W_L s/t = 98.1(840)/(2 \times 60)$

$= 687$ W

3.

$P = (W_L g)s/t = (3.5 \times 9.81) \times 4000/(6 \times 60) = 382$ W

5.

$P = 0$

An isometric contraction does no mechanical work.

7.

$$\omega = 200\frac{\text{deg}}{s} \times \frac{2\pi\ \text{rad}}{360\,\text{deg}} = 3.49$$

$P = M\omega = 50.0 \times 3.39 = 174.5$ W

9.

$$\omega = 3\frac{r}{s} \times \frac{2\pi\ \text{rad}}{1r} = 18.85\ \text{rad/s}$$

$P = M\omega = 55.0 \times 18.85 = 1037$ W

11.

$P = Fv = 225 \times 0.555 = 124.9$ W

$W = Pt = 124.9 \times 3.50 = 437$ J

CONSERVATION of MECHANICAL ENERGY (p. 178)

$$\boxed{E_f = E_i = \text{constant}}$$

1.

$$W_G = mgy = 75.0 \times 9.81 \times 3.00 = 2210 \text{ J}$$
$$\tfrac{1}{2}mv^2 = mgy$$
$$v = \sqrt{2gy} = \sqrt{2(9.81)3} = 7.67 \text{ m/s}$$

3.

$$E_f = E_i$$
$$\tfrac{1}{2}mv_f^2 + mgy_f = \tfrac{1}{2}mv_i^2 + mgy_i$$
$$\tfrac{1}{2}mv_f^2 + 0 = \tfrac{1}{2}(65.0)4.15^2 + 65.0(9.81)10.00$$
$$= 559.7 + 6376.5 = 6936 \text{ J}$$
$$v_f = \sqrt{\frac{2 \times 6936}{65.0}} = \sqrt{213.4} = 14.61 \text{ m/s}$$

5.

$$mgy = \tfrac{1}{2}mv^2$$
$$y = \frac{v^2}{2g} = \frac{5.50^2}{2(9.81)} = \frac{30.25}{19.62} = 1.542 \text{ m}$$

7.

$$W = mgy_{top} = \tfrac{1}{2}mv_{takeoff}^2$$
$$W = 60.0(9.81)0.452 = 266 \text{ J}$$
$$\tfrac{1}{2}mv_{takeoff}^2 = 266 \text{ J}$$
$$v_{takeoff} = \sqrt{\frac{2 \times 266}{60.0}} = \sqrt{8.868} = 2.98 \text{ m/s}$$

9.

$$mgy = \tfrac{1}{2}mv^2$$
$$mgy = (15.00 + 0.100)(9.81)(0.35) = 51.85 \text{ J}$$
$$\tfrac{1}{2}mv^2 = 51.85 \text{ J}$$
$$v = \sqrt{\frac{2 \times 51.85}{0.100}} = \sqrt{1036.9} = 32.2 \text{ m/s}$$